

Day

Quiz 3

Your name:

Your TA's name:

Solutions

1. [10 points] Find the limit, probably infinite, if it exists. If the limit does not exist explain why.

$$\lim_{x \rightarrow 1^-} e^{\frac{5}{x-1}}$$

Denote $y = \frac{5}{x-1}$ $y \rightarrow -\infty$ as $x \rightarrow 1^-$

Then $\lim_{x \rightarrow 1^-} e^{\frac{5}{x-1}} = \lim_{y \rightarrow -\infty} e^y = \boxed{0}$

2. [10 points] Find the limit. Use l'Hospital's Rule if it is appropriate. Explain why you applied or could not apply l'Hospital's Rule.

$$\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1}$$

$$\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 1} \frac{ax^{a-1}}{bx^{b-1}} = \frac{a \cdot 1}{b \cdot 1} = \boxed{\frac{a}{b}}$$

3. [10 points] Use logarithmic differentiation to find y' if $y = x^{\cos x}$.

$$\ln y = \cos x \cdot \ln x$$

$$\text{DBS: } \frac{y'}{y} = -\sin x \cdot \ln x + \frac{\cos x}{x}$$

$$y' = x^{\cos x} \cdot \left(\frac{\cos x}{x} - \sin x \ln x \right)$$

4. [10 points] Find $(f^{-1})'(a)$ if $f(x) = x^2 + 1$ and $a = 1$.

$$f^{-1}(1) = x \quad \Leftrightarrow \quad f(x) = 1 \quad \Leftrightarrow \quad x^2 + 1 = 1 \\ \Rightarrow x = 0$$

Hence $f^{-1}(1) = 0$.

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)}$$

$$f'(x) = 2x \quad , \quad f'(0) = 0$$

Hence $(f^{-1})'(1)$ DNE.