E

Quiz 3 **Your name:** Solutions

Your TA's name:

1. **Find an equation of the tangent line to the curve** $\frac{y^2-x^2}{2x+3y} = -2x$ **at the point** $(-1,1)$.

$y^2+x^2 = -2x(2x+3y) = -4x^2-6xy$

DBS: $2yy'+2x = -8x-6y-6xy'$

Plug in $x=-1$, $y=1$

$2y'-2 = 8-6+6y'$

$2y'-6y' = 2+2$

$-4y' = 4$

$y' = -1 = m$

tan line: $y = 1-1(x-(-1))$

$y = 1-x-1$

$\boxed{y = -x}$

2. Find the derivative of the function $y = \sqrt[3]{\frac{4x-1}{3x+7}}$.

$$y = \left(\frac{4x-1}{3x+7}\right)^{1/3} = (4x-1)^{1/3} \cdot (3x+7)^{-1/3}$$

$$y' = \frac{1}{3} \cdot 4\,(4x-1)^{-2/3}(3x+7)^{-1/3} +$$

$$+ (4x-1)^{1/3}\left(-\frac{1}{3}\right) \cdot 3\,(3x+7)^{-4/3}$$

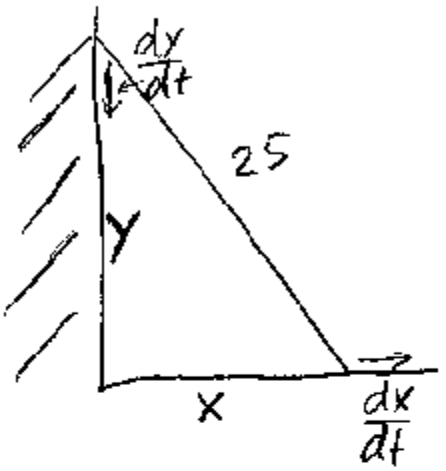
$$= \frac{1}{3} \cdot (4x-1)^{-2/3}(3x+7)^{-4/3} \cdot [4(3x+7) - 3(4x-1)] =$$

$$= \frac{12x+28-12x+3}{3\sqrt[3]{(4x-1)^2(3x+7)^4}} = \frac{31}{3\sqrt[3]{(4x-1)^2(3x+7)^4}}$$

Another way: $y' = \frac{1}{3}\left(\frac{4x-1}{3x+7}\right)^{-2/3} \cdot \frac{4(3x+7)-3(4x-1)}{(3x+7)^2}$

$$= \frac{1}{3}\left(\frac{3x+7}{4x-1}\right)^{2/3} \cdot \frac{31}{(3x+7)^2} = \frac{31}{3(4x-1)^{2/3}(3x+7)^{4/3}}$$

3. A 25-foot ladder is leaning against a vertical wall. If the base of the ladder is pulled horizontally away from the wall at a rate of 0.2 ft/sec, how fast is the top of the ladder sliding down that wall at the instant when it is 20 feet from the ground?



$$y^2 + x^2 = 25$$

$$y = 20, \quad \frac{dx}{dt} = 0.2$$

DBS: $$2y\frac{dy}{dt} + 2x\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$x = \sqrt{25^2 - 20^2} = 15$$

$$\frac{dy}{dt} = -\frac{15}{20} \cdot 0.2 = \boxed{-0.15 \text{ ft/sec}}$$