Quiz 5 **Your name:** Solutions

Your TA's name:

1. [10 points] Find the absolute maximum and the absolute minimum of the function $f(x) = \dfrac{x}{x^2+1}$ on the interval $[-3, 3]$.

$f(x)$ is continuous and differentiable on $[-3,3]$

crit. #'s : $f'(x) = \dfrac{x^2+1-2x\cdot x}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

$f'(x) = 0 \iff 1-x^2=0$, $x=-1$, $x=1$

$f'(x)$ exists everywhere

crit. #'s are $x=-1$ and $x=1$

$f(-3) = \dfrac{-3}{10} = -\dfrac{3}{10}$, $f(-1) = -\dfrac{1}{2} = -\dfrac{5}{10}$

$f(1) = \dfrac{5}{10}$, $f(3) = \dfrac{3}{10}$

Abs max value is $\frac{1}{2}$ at $x=1$

Abs min value is $-\frac{1}{2}$ at $x=-1$

2. By taking the natural logarithm if necessary and using the l'Hospital's Rule evaluate the limits

(a) [10 points] $\lim_{x\to 0} \frac{\cos^2 x - 1}{3x^2}$ $\underset{"\frac{0}{0}"}{\overset{H}{=}}$ $\lim_{x\to 0} \frac{2\cos x(-\sin x)}{6x}$ =

$$= -\lim_{x\to 0} \frac{\sin 2x}{6x} \underset{"\frac{0}{0}"}{\overset{H}{=}} -\lim_{x\to 0} \frac{2\cos 2x}{6} = -\frac{2}{6}$$

$$= -\frac{1}{3}$$

(b) [10 points] $\lim_{x \to 0} x^{\sqrt{x}}$

Let $y = x^{\sqrt{x}}$, then $\ln y = \sqrt{x} \ln x$

DBS: $\frac{y'}{y} = \frac{1}{2\sqrt{x}} \ln x + \sqrt{x} \cdot \frac{1}{x}$

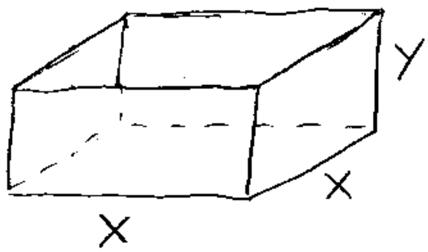
$$= \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$$

$$= \frac{\ln x + 2}{2\sqrt{x}}$$

$$y' = x^{\sqrt{x}} \cdot \frac{\ln x + 2}{2\sqrt{x}}$$

$$y' = \frac{1}{2} x^{\sqrt{x} - \frac{1}{2}} \cdot (\ln x + 2)$$

3. [10 points] A rectangular box with a square base and no top is to have a volume of 4 cubic inches. Find the dimensions for the box that require the least amount of material.



$V = x^2 y = 4$

$A = x^2 + 4xy \to \min$

$y = \frac{4}{x^2}$

$A = x^2 + 4x \cdot \frac{4}{x^2} = x^2 + \frac{16}{x} \to \min$

$A'(x) = 2x - \frac{16}{x^2} = \frac{2(x^3 - 8)}{x^2} = 0$

$\Leftrightarrow \quad x^3 - 8 = 0 \quad , \quad x = 2$

$A''(x) = 2 + \frac{32}{x^3} > 0$ since $x > 0$

Hence the func $A(x)$ is concave up when $x > 0$ $\Rightarrow$ it has only one abs. min at $x = 2$, $y = \frac{4}{x^2} = \frac{4}{2^2} = 1$

Answer: base is 2 in x 2 in
height is 1 in