# Exam 1

**Math 0220** (evening) **Spring 2011**

100 points total **Student's name:** _____

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1. Find the limit, if it exists. If the limit does not exist explain why. Show all the necessary steps. You can use any method.

(a) [5 points] $\lim_{x \to -2} \dfrac{\sqrt{x^2 - 4} - x}{2x}$

(b) [5 points] $\lim_{\theta \to \pi^-} \cot \theta$

(c) [5 points] $\lim_{h \to 2} \dfrac{h-2}{2|h-2|}$

(d) [5 points] $\lim_{x \to \infty} \dfrac{x \sin x}{x^2+1}$

2. [10 points] Calculate $y''$ if $y^2 - x^2 = 2x$. Do not leave $y'$ in your answer.

3. [15 points] A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 0.75 ft/sec. At what rate is the top of the ladder changing when the bottom of the ladder is 8 feet from the wall?

4. [15 points] Find an equation of the tangent line to the curve $y = \dfrac{\sqrt{x}}{x+1}$ at the point $(4, 0.4)$. Write the answer in the form $y = mx + b$.

5. [10 points] Find the derivative of the function $y = \sin t + \pi \cos(3t^2)$.

6. [10 points] Find the derivative of the function using the definition(!) of derivative (no credit will be given if you do not use the definition) if $f(x) = \dfrac{x^2}{2} - 1$.

7. Consider the function

$$f(x) = \begin{cases} \cos(\pi x), & \text{when} \quad 0 \leq x < \dfrac{1}{2} \\ -2x + 1, & \text{when} \quad \dfrac{1}{2} \leq x \leq 1 \end{cases}$$

(a) [10 points] Show that $f(x)$ is continuous on $[0, 1]$

(b) [10 points] Using the Intermediate Value Theorem prove that there is a number $c$ in the interval $(0, 1)$ such that $f(c) = \dfrac{1}{2}$.

Bonus problem. [10 points extra] A particle moves according to a law of motion $s = t^3 - 12t^2 + 36t$, where t is measured in seconds and s in feet, and $t \geq 0$. Find the total distance traveled during the first 8 seconds if $s(8) = 32$.