Final Exam

Math 0220 (evening) **Spring 2011**

100 points total **Student's name:** _____

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1. Evaluate the integrals.

(a) [10 points] $\int_0^{\pi/2} \cos^3 x \, dx$

(b) [5 points] $\int 8x\, e^{2x^2}\, dx$

2. [15 points] A rectangular sheet of paper with perimeter 36 cm is to be rolled into a cylinder. What are the dimensions of the sheet that give the greatest volume?

3. Find the limit, if it exists. If the limit does not exist explain why. Show all the necessary steps and justify your solution. You may use any method.

(a) [5 points] $\lim_{\theta \to 0^+} (1 + 3\theta)^{\cot \theta}$

(b) [5 points] $\lim_{t \to -2} \dfrac{t+2}{3|t+2|}$

(c) [5 points] $\lim_{x \to \infty} \dfrac{\cos(2x^2)}{x}$

4. (a) [5 points] Find the derivative of the function $g(x) = \int_2^{\sqrt{x}} \sqrt{t^2 - 1}\, dt$

(b) [10 points] Find an equation of the tangent line to the curve $y = \ln(2\tan x)$ when $x = \pi/4$. Write the answer in the form $y = mx + b$. No full credit if the form is different.

5. For the function $f(x) = \dfrac{e^x}{x}$

(a) [2 points] Find its domain.

(b) [5 points] Find vertical and horizontal asymptotes (if any).

(c) [5 points] Find the intervals on which f is increasing or decreasing.

(d) [3 points] Find the local maximum and minimum values of f (f, not x!)

(e) [5 points] Find the intervals of concavity [Hint: $x^2 - 2x + 2 > 0$ for any x].

6. (a) [15 points] Find the average value f_{ave} of the function $f = x \sin \pi x$ on the interval $[-1, 1]$.

(b) [5 points] Is there a number c inside the interval $(-1, 1)$ such that $f(c) = f_{ave}$? Support your answer.