Quiz 3 **Student's name:** Solutions

Math 0220 (evening) Spring 2011 **TA's name:** ____________

1. [5 points] Use linear approximation to estimate $(8.06)^{2/3}$.

To find the linear approximation consider the function $f(x) = x^{2/3}$. The "good point" is $a=8$, since $f(8)=4$.

The tan. line at 8 gives the linear approximation.

Find slope: $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$, $f'(8) = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$

Then tan. line is $y = f(8) + f'(8)(x-8)$

$y = 4 + \frac{1}{3}(x-8)$ (lin. approximation, i.e. $f(x) \approx 4 + \frac{1}{3}(x-8)$ near 8)

Then $(8.06)^{2/3} = f(8.06) \approx 4 + \frac{1}{3}(8.06-8) =$

$= 4 + \frac{1}{3}\cdot 0.06 = 4 + 0.02 = \boxed{4.02}$

2. [5 points] Find the limit, finite or infinite $\lim_{x \to -\infty} e^{-x^2}$.

$$\lim_{x \to -\infty} e^{-x^2} = \lim_{t \to \infty} e^{-t} = 0 \quad \text{(known fact)}$$

where $t = x^2$ (>0)

3. [5 points] If $h(x) = e^x + 2x + 7$ find $h^{-1}(8)$.

Note, $h(0) = e^0 + 2 \cdot 0 + 7 = 1 + 7 = 8$.

Hence $h^{-1}(8) = \boxed{0}$ by the definition of the inverse function.