Quiz 4 **Student's name:** Solutions

Math 0220 (evening) Spring 2011 **TA's name:** _____

1. [5 points] Differentiate the function $y = e^{x \cos x}$.

Using Chain Rule and Product Rule

$$y' = e^{x\cos x} \cdot (x \cdot \cos x)' = e^{x\cos x} \cdot (\cos x + x(-\sin x))$$

$$\boxed{y' = e^{x\cos x}(\cos x - x\sin x)}$$

2. [5 points] Find the limit. Use l'Hospital's Rule if it is necessary.

$\lim_{x\to 0}(\csc x - \cot x)$.

The limit is of the form "$\infty - \infty$".

$$\lim_{x\to 0}(\csc x - \cot x) = \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\frac{1-\cos x}{\sin x} \overset{H}{=} \text{"}\frac{0}{0}\text{"} \lim_{x\to 0}\frac{\sin x}{\cos x} = \frac{0}{1} = \boxed{0}$$

3. [5 points] A bacteria culture initially contains 100 cels and grows at a rate proportional to its size. After an hour the population has increased to 300. Find the number of bacteria after t hours.

The law is $P(t) = P_0 e^{kt}$

We have $P_0 = 100$

$P(1) = 300$

$P(t) = 100\, e^{kt} \Rightarrow P(1) = 100\, e^{k}$

Hence $100\, e^{k} = 300 \Rightarrow e^{k} = 3$

(No need to find k !)

Then $P(t) = 100 \cdot (e^{k})^{t} = \boxed{100 \cdot 3^{t}}$