

11-11:50am

Midterm Exam 1

Fall 2012

Math 0220

100 points total

Your name: _____

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

1. (10 points) Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{3x - 15}$$

In your work mention what Rules, Laws, Theorems or Formulas you use.

2. (10 points) Find the first and second derivatives of the function

$$g(x) = \sqrt{2x^2 - 1}$$

Simplify your answer. In your work mention what Rules, Laws, Theorems or Formulas you use.

3. (10 points) Use implicit differentiation to find an equation of the tangent line to the curve $x^{2/3} + y^{2/3} = 5$ at the point $(8, 1)$. Write the answer in the slope-intercept form.

4. (10 points) Sketch the graph of an example of a function $g(x)$ if it satisfies all the given conditions

$$g(0) = -2, \quad g'(0) = 1, \quad \lim_{x \rightarrow 1^-} g(x) = -1, \quad \lim_{x \rightarrow 1^+} g(x) = 1, \quad g(1) = 0$$
$$g(2) = 1, \quad g'(2) = -1, \quad g'(5) = 1 \quad \text{and} \quad \lim_{x \rightarrow \infty} g(x) = 0.$$

Mark all the essential points on the axes.

5. (10 points) Use the Intermediate Value Theorem to show that there is root of the equation $4 - x^2 = \sin x$ in the interval $(0, \pi)$. Support every step of your proof.

6. (10 points) Find all horizontal asymptotes of the curve

$$y = \frac{\sqrt{4x^2 + 2}}{2x + 6}$$

Justify your answer by calculating corresponding limits. [Use $\sqrt{x^2} = -x$ when $x < 0$.]

7. (15 points) A camera is located 40 feet away from a straight road along which a car is traveling with a constant speed of 90 feet per second. The camera turns so that it is pointed at the car at all times. In radians per second, how fast is the camera turning as the car passes closest to the camera?

8. (10 points) Use a linear approximation to estimate the number $\sqrt{4.99}$.

9. (15 points) Use differentials to estimate the amount of paint needed to apply a coat of paint 0.02 cm thick to a hemispherical dome with diameter 120 cm.

bonus problem [10 points extra] Find the 21st derivative $f^{(21)}(x)$ of the function $f(x) = \sin^2 x - \cos^2 x$.