12pm class

Quiz 1

Fall 2012 **Your name:** Answer key

Math 0220 **Your TA's name:** _____

No calculators. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. [5 points] Calculate the volume V of the cone with the radius $r = 3$ in. and the hight $h = 5$ in.

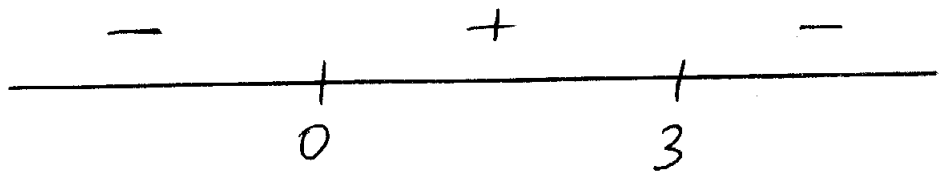
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 9 \cdot 5 = 15\pi \text{ in}^3$$

2. Solve inequalities

(a) [5 points] $3x - x^2 < 0$

$x(3-x) = 0$ has two roots 0 and 3.

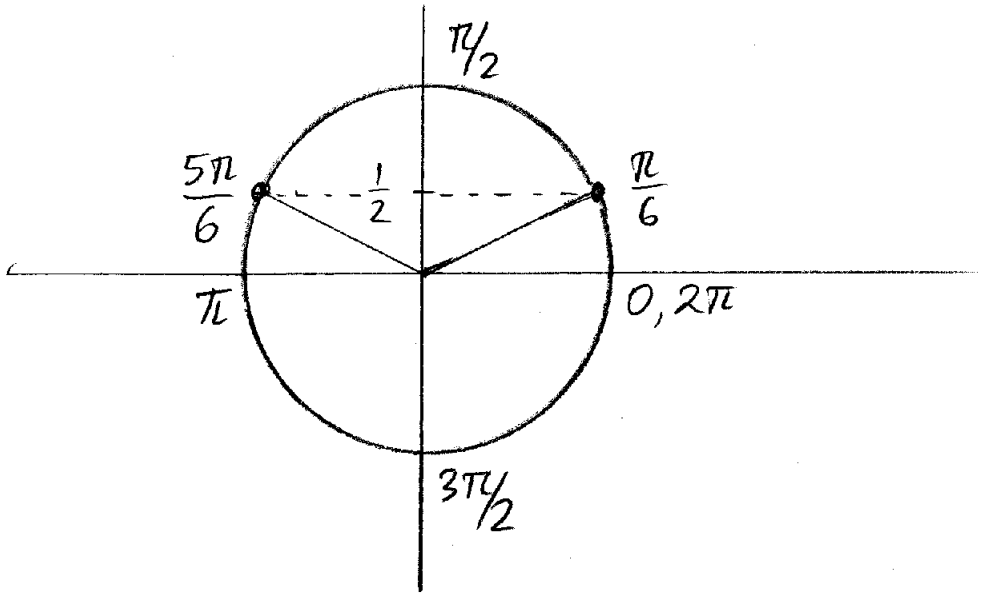
We use the method of intervals:



Answer: $x \in (-\infty, 0) \cup (3, \infty)$

(b) [5 points] $\sin\theta > \frac{1}{2}$ on the interval $0 \le \theta \le 2\pi$.

$\sin\frac{\pi}{6} = \frac{1}{2}$ and $\sin\frac{5\pi}{6} = \frac{1}{2}$



Hence $\sin\theta > \frac{1}{2}$ when $\theta \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

3. [5 points] One of the roots of the polynomial $p(x) = x^3 + 3x^2 - 5x + 1$ is $x_1 = 1$. Find the other roots or show that the polynomial does not have more roots.

Since $x=1$ is the root then $p(x) = (x-1)g(x)$. Use long division for polynomials to find $g(x)$:

$$
\begin{array}{r}
x^2+4x-1 \\
\hline
x-1 \;\big|\; x^3+3x^2-5x+1 \\
x^3-x^2 \\
\hline
4x^2-5x+1 \\
4x^2-4x \\
\hline
-x+1 \\
-x+1 \\
\hline
0
\end{array}
$$

Hence $p(x) = (x-1)(x^2+4x-1)$. To find roots of x^2+4x-1 we use the quadratic formula: $x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm 2\sqrt{5}}{2}$

$x_2 = -2-\sqrt{5}$

$x_3 = -2+\sqrt{5}$

bonus problem [5 points extra] Calculate $\sin\left(\frac{5\pi}{12}\right)$. Write the answer in form of radicals. Hint: $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$.

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right).$$