11am

Quiz 2

Fall 2012 **Your name:** Solutions

Math 0220 **Your TA's name:** _____

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. (a) [4 points] The functions $F(x) = \sec\left((x^2-4)^{-1/3}\right)$ can be expressed in the form $f \circ g \circ h$. Find $f(x)$, $g(x)$, and $h(x)$.

$f(x) = \sec x$, $g(x) = x^{-1/3}$, $h(x) = x^2 - 4$

(b) [3 points] What is the domain of $g \circ h$?

$g \circ h = g(h(x)) = (x^2-4)^{-1/3} = \dfrac{1}{\sqrt[3]{x^2-4}}$

Domain: $x^2 - 4 \neq 0$, $x \neq -2$, $x \neq 2$

Answer $D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

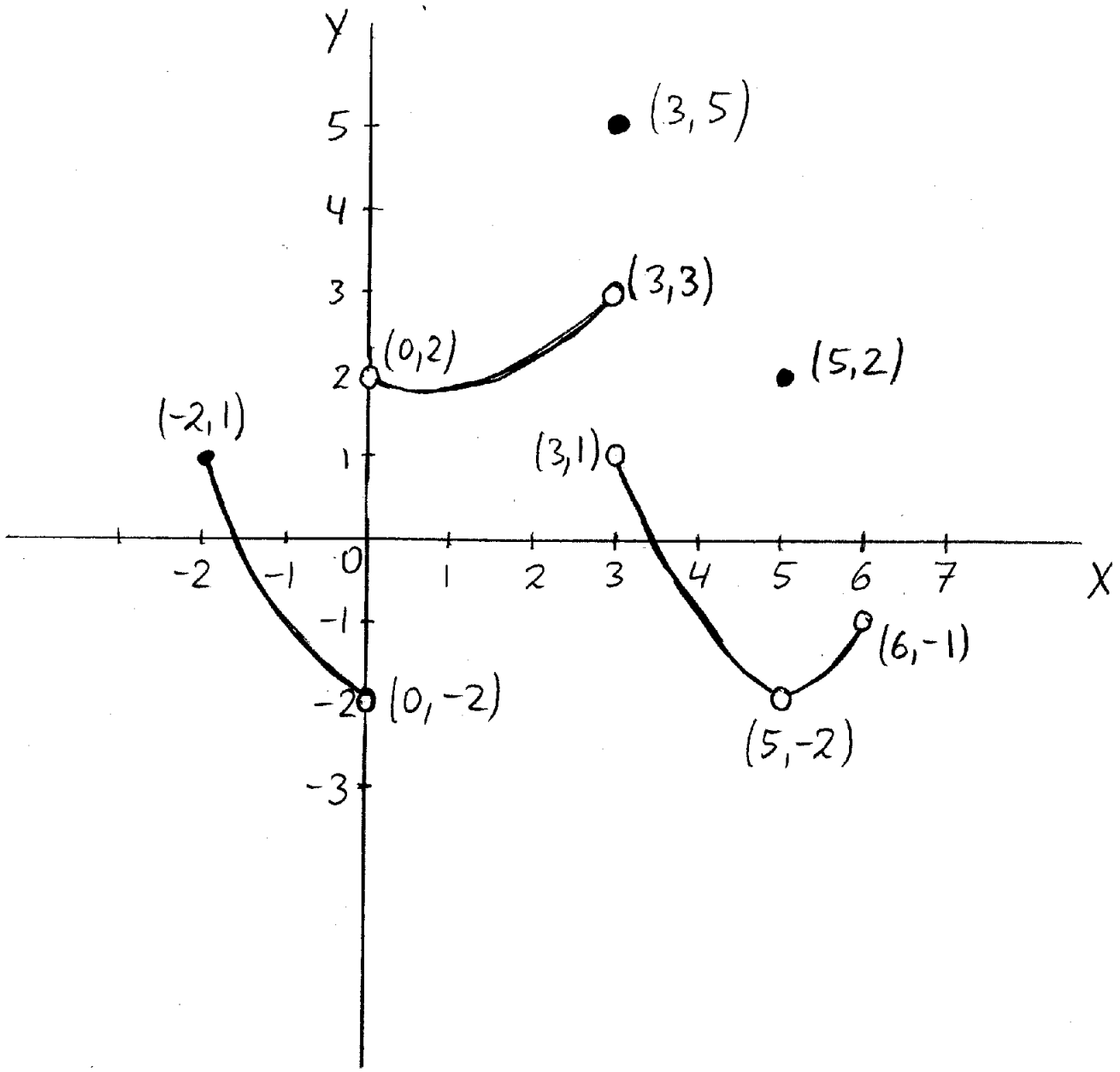
(Note: the root has odd degree $\Rightarrow$ $x^2 - 4$ can be negative)

2. [7 points] Sketch the graph of an example of a function $g(x)$ if it has the domain $[-2, 6)$ and satisfies all the given conditions. Mark all important points on the graph and the axes.

$g(-2) = 1$, $\lim_{x \to 0^-} g(x) = -2$, $\lim_{x \to 0^+} g(x) = 2$, $g(2)$ is undefined,

$g(3) = 5$, $\lim_{x \to 3^-} g(x) = 3$, $\lim_{x \to 3^+} g(x) = 1$,

$\lim_{x \to 5} g(x) = -2$, $g(5) = 2$, $\lim_{x \to 6^-} g(x) = -1$.



3. [6 points] Evaluate the difference quotient $\frac{f(2+h)-f(2)}{h}$ for the function $f(x)=x^3$.

$$\frac{f(2+h)-f(2)}{h} = \frac{(2+h)^3-2^3}{h}$$

$$= \frac{2^3+3\cdot 4h+3\cdot 2h^2+h^3-2^3}{h}$$

$$= \frac{12h+6h^2+h^3}{h}$$

$$= 12+6h+h^3.$$

bonus problem [5 points extra] How big do you think the limit

$$\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2}$$

is?

$$\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - x - 2} = \lim_{x \to 2} \frac{x(x-2)}{(x-2)(x+1)}$$

$$= \lim_{x \to 2} \frac{x}{x+1} = \frac{2}{2+1} = \frac{2}{3}$$