12pm

Quiz 2

Fall 2012 **Your name:** Solutions

Math 0220 **Your TA's name:** _____

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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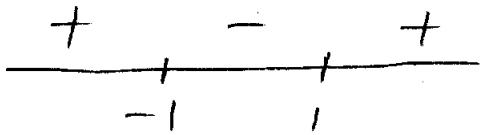
1. (a) [4 points] What is the domain of $f(x) = \dfrac{\sqrt{x^2-1}}{\cos x}$ inside the interval $[0, \pi]$?

$x^2 - 1 \geq 0 \iff (x-1)(x+1) \geq 0$

$x \leq -1$ or $x \geq 1$



inside $[0, \pi]$: $x \geq 1$

$\cos x \neq 0$ when $x \neq \frac{\pi}{2}$ inside $[0, \pi]$.

Domain: $D = [1, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

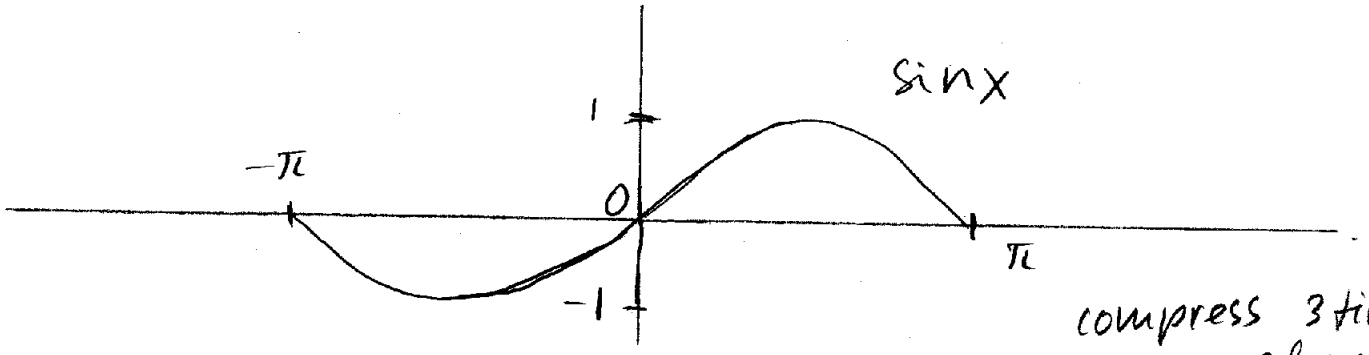
or $x \in [1, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

(b) [3 points] Determine whether the function in the part (a) $\left(f(x) = \dfrac{\sqrt{x^2-1}}{\cos x}\right)$ is even, odd or neither inside its domain.

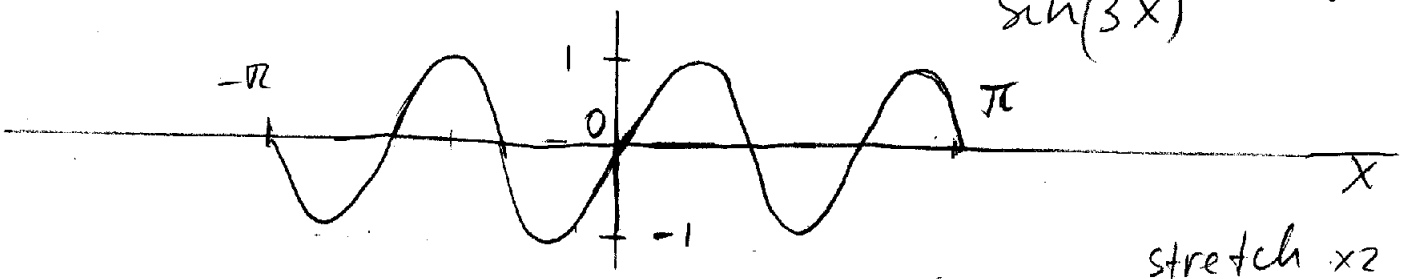
$$f(-x) = \frac{\sqrt{(-x)^2-1}}{\cos(-x)} = \frac{\sqrt{x^2-1}}{\cos x} = f(x)$$

Hence $f(x)$ is even.

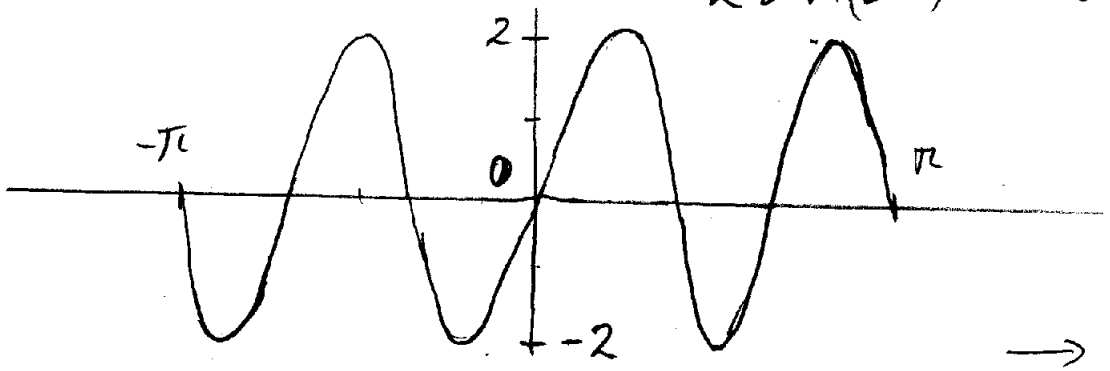
2. [6 points] Starting from the graph of $\sin x$ sketch the graph of the function $g(x) = 2\sin\left(3x - \frac{\pi}{2}\right)$. Mark all important points on the axes.



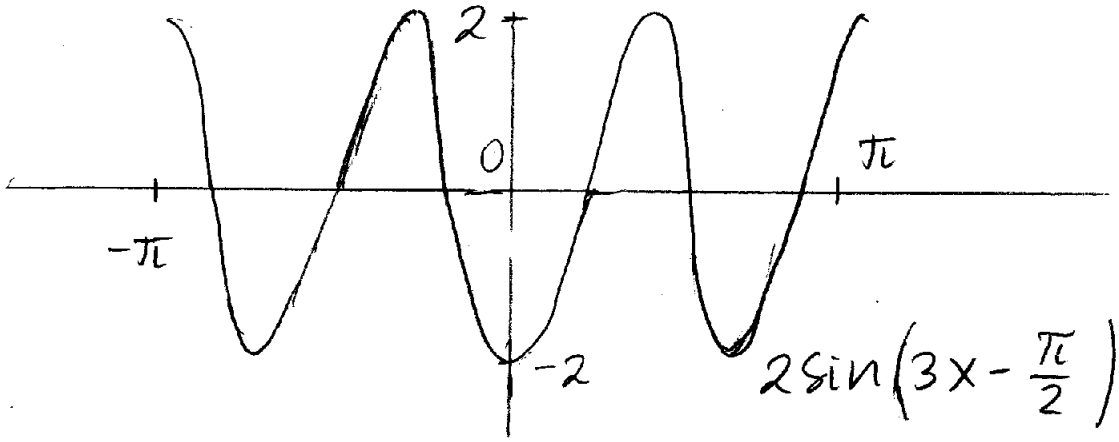
compress 3 times along x



stretch x2 along y



→ $\frac{\pi}{2}$ right shift



$2\sin\left(3x - \frac{\pi}{2}\right)$

3. (a) [3 points] Find the function $g \circ f$ if

$$f(x) = (x-2)^2, \quad g(x) = \sqrt{x-1}.$$

Simplify your answer.

$g \circ f = g(f(x)) = \sqrt{f(x)-1} = \sqrt{(x-2)^2 - 1}$

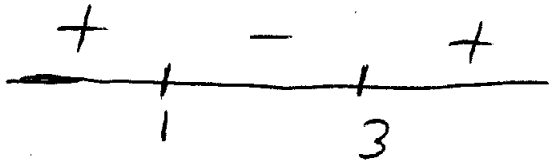
$= \sqrt{x^2-4x+4-1} = \sqrt{x^2-4x+3}$

(b) [4 points] Find the domain (maximal possible) of the function $g \circ f$.

Domain : $x^2 - 4x + 3 \geq 0$

$x^2-4x+3 = (x-1)(x-3) \geq 0$

roots $x=1$, $x=3$



$x \in (-\infty, 1] \cup [3, \infty) = D$

bonus problem [5 points extra] Evaluate the difference quotient $\frac{f(x)-f(1)}{x-1}$ for the function $f(x)=\frac{x}{x+1}$.

$$\frac{f(x)-f(1)}{x-1} = \frac{\frac{x}{x+1}-\frac{1}{1+1}}{x-1} = \frac{\frac{x}{x+1}-\frac{1}{2}}{x-1}$$

$$= \frac{\frac{2x-x-1}{2(x+1)}}{x-1} = \frac{x-1}{2(x+1)(x-1)} = \frac{1}{2(x+1)}$$