11am

Quiz 3

Fall 2012 **Your name:** Solutions

Math 0220 **Your TA's name:** ____________

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. [10 points] Evaluate the limit $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{x}$, if it exists.

$$\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} \cdot \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$$

$$= \lim_{x\to 0} \frac{4+x-4}{x(\sqrt{4+x}+2)} = \lim_{x\to 0} \frac{x}{x(\sqrt{4+x}+2)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{\sqrt{\lim\limits_{x\to 0}(4+x)}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

2. [10 points] Is the function

$$f(x) = \begin{cases} \cos x & \text{if} \quad x < \pi/4 \\ \sin x & \text{if} \quad \pi/4 \le x < \pi/2 \\ \cos x & \text{if} \quad x \ge \pi/2 \end{cases}$$

continuous on $(-\infty, \infty)$. If it is not, then find points at which it is discontinuous. Provide the full explanation.

The function $f(x)$ is continuous on $(-\infty, \frac{\pi}{4}) \cup (\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \infty)$ since it is either $\sin x$ or $\cos x$ there. We need to check continuity at $\frac{\pi}{4}$ and $\frac{\pi}{2}$:

$$\left.\begin{aligned} \lim_{x\to\frac{\pi}{4}^-} f(x) &= \lim_{x\to\frac{\pi}{4}^-} \cos x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \\ \lim_{x\to\frac{\pi}{4}^+} f(x) &= \lim_{x\to\frac{\pi}{4}^+} \sin x = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \Rightarrow \lim_{x\to\frac{\pi}{4}} f(x) = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4} = f\left(\frac{\pi}{4}\right)$$

$\Rightarrow f(x)$ is continuous at $\frac{\pi}{4}$

$$\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^-} \sin x = \sin\frac{\pi}{2} = 1$$

$$\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to\frac{\pi}{2}^+} \cos x = \cos\frac{\pi}{2} = 0 \ne \lim_{x\to\frac{\pi}{2}^-} f(x)$$

Hence $f(x)$ is not continuous on $(-\infty, \infty)$.
It is discontinuous at $\frac{\pi}{2}$.

bonus problem [5 points extra] Does the function

$$f(x) = \frac{x^3 + 27}{x + 3}$$

have a removable discontinuity at $x = -3$? If the discontinuity is removable, then find a continuous everywhere function g that agrees with f for $x \neq -3$.

$$\lim_{x \to -3} \frac{x^3+27}{x+3} = \lim_{x \to -3} \frac{(x+3)(x^2-3x+9)}{x+3}$$

$$= \lim_{x \to -3} (x^2-3x+9) = (-3)^2 - 3(-3) + 9$$

$$= 27$$

The limit exists $\Rightarrow$ $f(x)$ has removable discontinuity at -3.

$$g(x) = \begin{cases} \frac{x^3+27}{x+3} & x \neq -3 \\ 27 & x = -3 \end{cases}$$