12pm

Quiz 3

Fall 2012 **Your name:** Solutions

Math 0220 **Your TA's name:** _____

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. [8 points] Find the limit $\lim_{x\to 0^-} x^3 \sin\frac{3}{x}$.

$-1 \le \sin\frac{3}{x} \le 1$ (multiply by x^3)

$-x^3 \ge x^3 \sin\frac{3}{x} \ge x^3$ (because $x^3 < 0$)

or $x^3 \le x^3 \sin\frac{3}{x} \le -x^3$

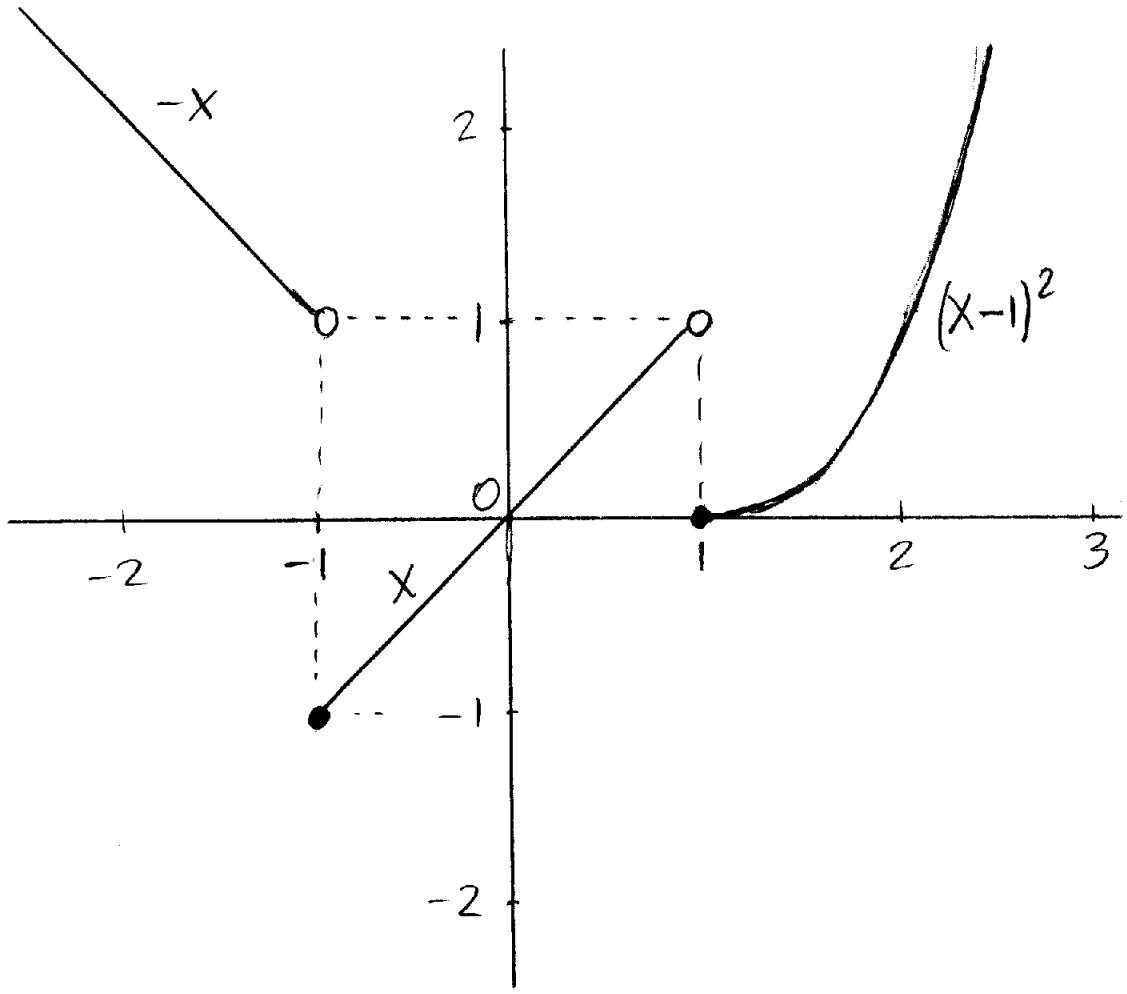
$\lim_{x\to 0^-} x^3 = 0$, $\lim_{x\to 0^-} (-x^3) = 0$.

By the Squeeze Theorem

$\lim_{x\to 0^-} x^3 \sin\frac{3}{x} = 0$

2. (a) [8 points] Sketch the graph of the function

$$f(x) = \begin{cases} -x & \text{if} \quad x < -1 \\ x & \text{if} \quad -1 \le x < 1 \\ (x-1)^2 & \text{if} \quad x \ge 1 \end{cases}$$



(b) [4 points] Use the graph to determine the values of a for which $\lim_{x \to a} f(x)$ exists. In your answer use the interval notation.

$$\left.\begin{aligned} \lim_{x \to -1^-} f(x) &= \lim_{x \to -1^-} (-x) = 1 \\ \lim_{x \to -1^+} f(x) &= \lim_{x \to -1^+} (x) = 1 \end{aligned}\right\} \Rightarrow \lim_{x \to -1} f(x) \text{ DNE}$$

$$\left.\begin{aligned} \lim_{x \to 1^-} f(x) &= \lim_{x \to 1^-} (x) = 1 \\ \lim_{x \to 1^+} f(x) &= \lim_{x \to 1^+} (x-1)^2 = 0 \end{aligned}\right\} \Rightarrow \lim_{x \to 1} f(x) \text{ DNE}$$

Hence $\lim_{x \to a} f(x)$ exists when

$$a \in (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

bonus problem [5 points extra] Find the limit $\lim_{t\to 0} \dfrac{5t\cot 3t}{\cos 2t}$.

$$\lim_{t\to 0} \frac{5t\cot 3t}{\cos 2t} = \lim_{t\to 0} \frac{5t}{\cos 2t}\cdot\frac{\cos 3t}{\sin 3t}$$

$$= \lim_{t\to 0} \frac{\cos 3t}{\cos 2t}\cdot\frac{5\cdot 3t}{3\sin 3t}$$

$$= \frac{5}{3}\lim_{t\to 0} \frac{\cos 3t}{\cos 2t}\cdot\lim_{t\to 0}\frac{3t}{\sin 3t}$$

$$\lim_{t\to 0} \frac{\cos 3t}{\cos 2t} \overset{\text{DSP}}{=} \frac{\cos 0}{\cos 0} = \frac{1}{1} = 1$$

$$\lim_{t\to 0} \frac{3t}{\sin 3t} \left[\begin{array}{l}\text{sub: } x = 3t \\ x\to 0 \text{ as } t\to 0\end{array}\right] = \lim_{x\to 0}\frac{x}{\sin x}$$

$$= \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} = \frac{1}{\lim\limits_{x\to 0}\frac{\sin x}{x}} = \frac{1}{1} = 1$$

Hence $\lim\limits_{t\to 0} \dfrac{5t\cot 3t}{\cos 2t} = \dfrac{5}{3}\cdot 1\cdot 1 = \dfrac{5}{3}$