11am

Quiz 4

Fall 2012 **Your name:** Solutions

Math 0220 **Your TA's name:** ______

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. Evaluate the limits (a) [3 points] $\lim_{x\to -4^+} \frac{x+3}{x+4}$

$\lim_{x\to -4^+} (x+3) = -1 < 0$

$x+4$ is getting arbitrary small and positive when $x \to -4^+$

Hence $\frac{1}{x+4}$ is getting arbitrary large and positive when $x\to -4^+$

$\lim_{x\to -4^+} \frac{x+3}{x+4} = -\infty$, because $x+3<0$ and $\frac{1}{x+4}$ is arbitrary large and positive

(b) [3 points] $\lim_{x\to\infty} \frac{\sin^2 x}{x^2}$

$-1 \le \sin x \le 1 \quad \Rightarrow \quad 0 \le \sin^2 x \le 1$

$$\Rightarrow \quad \frac{0}{x^2} \le \frac{\sin^2 x}{x^2} \le \frac{1}{x^2}$$

$$\text{or} \quad 0 \le \frac{\sin^2 x}{x^2} \le \frac{1}{x^2}$$

$$\lim_{x\to\infty} 0 = 0, \quad \lim_{x\to\infty} \frac{1}{x^2} = 0$$

$$\lim_{x\to\infty} 0 = \lim_{x\to\infty} \frac{1}{x^2} = 0$$

By the Squeeze Theorem

$$\lim_{x\to\infty} \frac{\sin^2 x}{x^2} = 0.$$

2. [4 points] Find an equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(4, 2)$.

Let $y = f(x)$, $f(x) = \sqrt{x}$.

$$m = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

$$= \lim_{h \to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{4+h}+2)} = \lim_{h \to 0} \frac{1}{\sqrt{4+h}+2}$$

$$\overset{DSP}{=} \frac{1}{\sqrt{4+0}+2} = \frac{1}{4}$$

tan. line eq. $y = 2 + \frac{1}{4}(x-4)$

$$\boxed{y = \frac{1}{4}x + 1}$$

bonus problem [5 points extra] Using the definition of derivative find the derivative of the function $f(x) = |x|$ on $(-\infty, 0) \cup (0, \infty)$.

$$f(x) = \begin{cases} x, & x \geqslant 0 \\ -x, & x < 0 \end{cases}$$

1. $x > 0$, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{x+h-x}{h} = 1$

2. $x < 0$, $f'(x) = \lim_{h \to 0} \frac{-(x+h) - (-x)}{h} = -1$

3. $x = 0$, the derivative from the left is -1, from the right is 1. They are not equal $\Rightarrow$ the derivative at 0 DNE.

$$f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$