12pm

Quiz 4

Fall 2012 **Your name:** Solutions

Math 0220 **Your TA's name:** ______________

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. [4 points] Find an equation of the tangent line to the curve $y = \dfrac{x-2}{x-1}$ at the point $(2, 0)$.

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}, \text{ where } f(x) = \frac{x-2}{x-1}$$

$$f(2+h) = \frac{2+h-2}{2+h-1} = \frac{h}{h+1}, \quad f(2) = 0$$

$$m = \lim_{h \to 0} \frac{\frac{h}{h+1}}{h} = \lim_{h \to 0} \frac{h}{h(h+1)}$$

$$= \lim_{h \to 0} \frac{1}{h+1} = \frac{1}{0+1} = 1$$

tan. line: $y = 0 + 1(x-2)$

$$\boxed{y = x - 2}$$

2. (a) [4 points] Does the function $f(x) = \dfrac{\sqrt{x+10}-3}{x+1}$ have removable discontinuity at $x=-1$? Support your answer.

$f(x)$ is undefined at $x=-1$
($x=-1$ is not in the domain of f).
Hence f is discontinuous at -1.

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{\sqrt{x+10}-3}{x+1} \cdot \frac{\sqrt{x+10}+3}{\sqrt{x+10}+3}$$

$$= \lim_{x\to -1} \frac{x+10-9}{(x+1)(\sqrt{x+10}+3)} = \lim_{x\to -1} \frac{x+1}{(x+1)(\sqrt{x+10}+3)}$$

$$= \lim_{x\to -1} \frac{1}{\sqrt{x+10}+3} \overset{\text{DSP}}{=} \frac{1}{\sqrt{-1+10}+3} = \frac{1}{6}$$

The limit at -1 exists.
Hence the discontinuity at -1 is removable.

(b) [2 points] If the discontinuity is removable find a continuous function $g(x)$ such that $f(x) = g(x)$ everywhere except at $x = -1$.

$$g(x) = \begin{cases} \dfrac{\sqrt{x+10} - 3}{x+1} & x \neq -1 \\ \dfrac{1}{6} & x = -1 \end{cases}$$

bonus problem [5 points extra] Find the limit $\lim_{t\to\infty}\left(\sqrt{t^2-at}-\sqrt{t^2-bt}\right)$.

$$\lim_{t\to\infty}\left(\sqrt{t^2-at}-\sqrt{t^2-bt}\right)$$

$$=\lim_{t\to\infty}\left(\sqrt{t^2-at}-\sqrt{t^2-bt}\right)\cdot\frac{\sqrt{t^2-at}+\sqrt{t^2-bt}}{\sqrt{t^2-at}+\sqrt{t^2-bt}}$$

$$=\lim_{t\to\infty}\frac{t^2-at-t^2+bt}{\sqrt{t^2-at}+\sqrt{t^2-bt}}$$

$$=\lim_{t\to\infty}\frac{(b-a)t}{\sqrt{t^2-at}+\sqrt{t^2-bt}}\cdot\frac{1/t}{1/t}$$

$$=\lim_{t\to\infty}\frac{(b-a)}{\sqrt{1-\frac{a}{t}}+\sqrt{1-\frac{b}{t}}}=\frac{b-a}{\sqrt{1-0}+\sqrt{1-0}}$$

$$=\boxed{\frac{b-a}{2}}$$

Note $\sqrt{t^2-at}\cdot\frac{1}{t}=\sqrt{\frac{t^2-at}{t^2}}=\sqrt{1-\frac{a}{t}}$

when $t>0$