11am

# Quiz 5

Fall 2012

**S o l u t i o n s**

Math 0220

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1. Differentiate

(a) [1 point] $f(x) = \sin x + x^2$

Solution: $f'(x) = \cos x + 2x$.

(b) [2 point] $y = \sqrt{\cos(x^2)}$

Solution: Chain rule: $y' = \dfrac{1}{2\sqrt{\cos(x^2)}}\left(-\sin(x^2)\right)(2x) = -\dfrac{x\sin(x^2)}{\sqrt{\cos(x^2)}}$.

(c) [2 points] $f(\theta) = \dfrac{\sec\theta}{1-\sec\theta}$

Solution: Qoutient rule: $f'(\theta) = \dfrac{(\sec\theta)'(1-\sec\theta) - \sec\theta(1-\sec\theta)'}{(1-\sec\theta)^2}$

$$= \frac{(\sec\theta\tan\theta)(1-\sec\theta) - \sec\theta(-\sec\theta\tan\theta)}{(1-\sec\theta)^2} = \frac{\sec\theta\tan\theta - \sec^2\theta\tan\theta + \sec^2\theta\tan\theta}{(1-\sec\theta)^2}$$

$$f'(\theta) = \frac{\sec\theta\tan\theta}{(1-\sec\theta)^2}.$$

2. [5 points] Find an equation of the tangent line to the curve

$y = \sin x - 2\sin^2 x$ at the point $(\pi, 0)$. Simplify your answer.

Solution: Tangent line equation at $(\pi, 0)$ is $y = 0 + m(x - \pi)$ or $y = mx - m\pi$.

$m = y'(\pi)$, $y' = \cos x - 2 \cdot 2 \sin x \cos x$, $y'(\pi) = -1 - 0 = -1$, because $\cos \pi = -1$ and $\sin \pi = 0$.

Hence, $m = -1$ and the tangent line equation is $y = -x + \pi$.

bonus problem [5 points extra] Find an equation of the tangent line to the

curve $y = \dfrac{|x|}{\sqrt{2 - x^2}}$ at the point $(-1, 1)$.

Solution: Tangent line equation at $(-1, 1)$ is $y = 1 + m(x - (-1))$ or $y = mx + m + 1$.

$m = y'(-1)$. Near the point $x = -1$ $x$ is negative $(x < 0)$ and $|x| = -x$. Then

$y = \dfrac{-x}{\sqrt{2 - x^2}} = -\dfrac{x}{\sqrt{2 - x^2}}$. To find the derivative we apply the qoutient and chain rules:

$$y' = -\frac{\sqrt{2 - x^2} - x\dfrac{-2x}{2\sqrt{2 - x^2}}}{2 - x^2}, \qquad y'(-1) = -\frac{1 - (-1)\dfrac{(-2)(-1)}{2 \cdot 1}}{1} = -2.$$

Hence, $m = -2$ and the tangent line equation is $y = -2x - 1$.