12pm

Quiz 5

Fall 2012 **S o l u t i o n s**

Math 0220

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1. Differentiate

(a) [1 point] $V(r) = \frac{4}{3}\pi r^3$

Solution: $V'(r) = \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2$.

(b) [1 point] $y = 5\pi^2$

Solution: $y' = 0$.

(c) [3 points] $f(t) = \dfrac{1 - \sin t}{\cos t}$

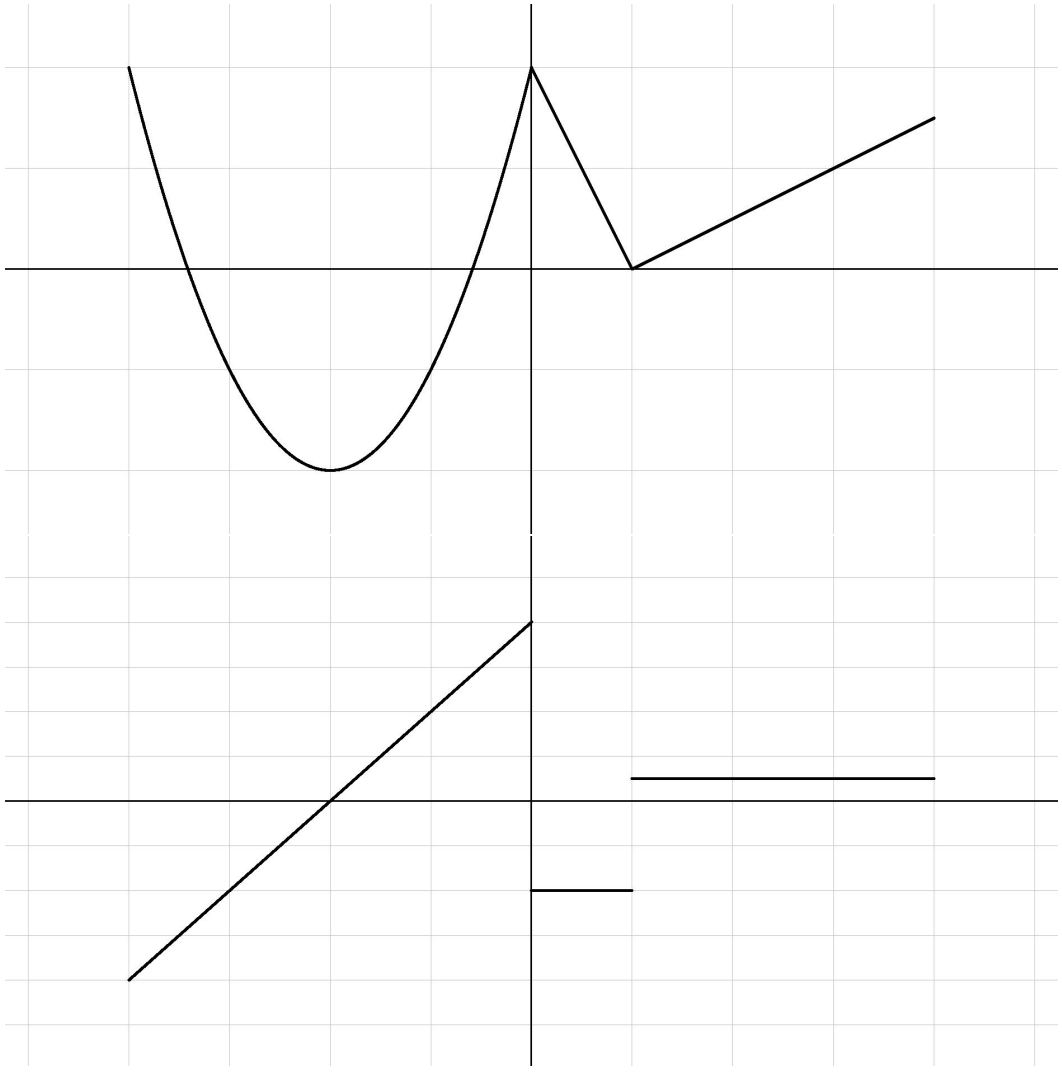
Solution: $f'(t) = \dfrac{(1 - \sin t)' \cos t - (1 - \sin t)(\cos t)'}{\cos^2 t}$

$= \dfrac{(-\cos t)\cos t - (1 - \sin t)(-\sin t)}{\cos^2 t}$

$= \dfrac{-\cos^2 t + \sin t - \sin^2 t}{\cos^2 t}$

$= \dfrac{\sin t - 1}{\cos^2 t}$.

2. [5 points] The graph of the function $f(x)$ is given. Sketch the graph of its derivative $f'(x)$ below the graph of $f(x)$.



bonus problem [5 points extra] Find an equation of the tangent line to the curve $y = (1 + x)\cos x$ when $x = \dfrac{\pi}{4}$. Simplify your answer. Write the equation in the slope-intercept form.

Solution: $y' = (1+x)'\cos x + (1+x)(\cos x)' = \cos x + (1+x)(-\sin x) = \cos x - (1+x)\sin x,$

$$m = y'(\pi/4) = \frac{\sqrt{2}}{2} - \left(1 + \frac{\pi}{4}\right)\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\pi}{4} = -\frac{\pi\sqrt{2}}{8}$$

$$y(\pi/4) = \left(1 + \frac{\pi}{4}\right)\frac{\sqrt{2}}{2}$$

The tangent line equation is $y = \left(1 + \dfrac{\pi}{4}\right)\dfrac{\sqrt{2}}{2} - \dfrac{\pi\sqrt{2}}{8}\left(x - \dfrac{\pi}{4}\right) = \left(1 + \dfrac{\pi}{4}\right)\dfrac{\sqrt{2}}{2} - \dfrac{\pi\sqrt{2}}{8}x + \dfrac{\pi^2}{16}\dfrac{\sqrt{2}}{2}.$

$$y = -\frac{\pi\sqrt{2}}{8}x + \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4} + \frac{\pi^2}{16}\right).$$