11am **Quiz 6**

Fall 2012 **S o l u t i o n s**

Math 0220

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1. [5 points] Use logarithmic differentiation to find y' if $y = (\sin x)^x$.

Solution: $\ln y = \ln(\sin x)^x$, $\ln y = x \ln(\sin x)$, $\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}(x \ln(\sin x))$,

$\dfrac{y'}{y} = \ln(\sin x) + x \cdot \dfrac{\cos x}{\sin x}$, $y' = y\,(\ln(\sin x) + x \cot x)$,

$y' = (\sin x)^x\,(\ln(\sin x) + x \cot x)$.

2. [5 points] The half-life of cesium-137 is 30 years. If you have a 200-mg sample how much of the sample remains after 75 years? Simplify your answer. Leave only radicals in it (no logarithms, no exponents).

Solution: Let $m(t)$ be the mass of cesium-137 that remains after t years. Then

$m(t) = m(0)\,e^{kt}$, $m(30) = m(0)\,e^{30k} = \dfrac{1}{2}\,m(0)$.

The last equations gives $e^{30k} = \dfrac{1}{2}$ or $e^k = \left(\dfrac{1}{2}\right)^{1/30} = 2^{-1/30}$.

Hence, $e^{kt} = 2^{-t/30}$ and $m(t) = m(0)\,2^{-t/30} = 200 \cdot 2^{-t/30}$.

$m(75) = 200 \cdot 2^{-75/30} = \dfrac{200}{2^{2\frac{1}{2}}} = \dfrac{200}{4\sqrt{2}} = \dfrac{50}{\sqrt{2}} = \dfrac{50\sqrt{2}}{2} = 25\sqrt{2}$ mg.

bonus problem [5 points extra] Solve the equation $3^{2^x} = 2^{3^x}$. Leave logarithms in your answer.

Solution: $\log_3\left(3^{2^x}\right) = \log_3\left(2^{3^x}\right)$, $2^x = 3^x \log_3 2$, $\dfrac{2^x}{3^x} = \log_3 2$, $\left(\dfrac{2}{3}\right)^x = \log_3 2$,

$\log_{\frac{2}{3}}\left(\dfrac{2}{3}\right)^x = \log_{\frac{2}{3}} \log_3 2$,

$x = \log_{\frac{2}{3}} \log_3 2$. Another answer is $x = \log_{\frac{3}{2}} \log_2 3$.