12pm

Quiz 6

Fall 2012

Solutions

Math 0220

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1. [5 points] Solve the equation $e^{\ln x^2 + \ln 5} = 10$.

Solution: $e^{\ln x^2}\, e^{\ln 5} = 10, \quad x^2 \cdot 5 = 10, \quad x^2 = 2$. Two solutions: $x = -\sqrt{2}$ or $x = \sqrt{2}$.

2. [5 points] Use logarithmic differentiation to find y' if $y = \dfrac{\sqrt{x}\, e^{2x}}{(x^3-2)^8}$.

Solution: $\ln y = \ln\left(\dfrac{\sqrt{x}\, e^{2x}}{(x^3-2)^8}\right) = \ln\left(x^{1/2}\right) + \ln\left(e^{2x}\right) - \ln\left((x^3-2)^8\right)$.

$$\ln y = \frac{1}{2}\ln x + 2x - 8\ln(x^3-2).$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\frac{1}{2}\ln x + 2x - 8\ln(x^3-2)\right).$$

$$\frac{y'}{y} = \frac{1}{2x} + 2 - 8\cdot\frac{3x^2}{x^3-2}, \quad y' = y\left(\frac{1}{2x} + 2 - \frac{24x^2}{x^3-2}\right).$$

$$y' = \frac{\sqrt{x}\, e^{2x}}{(x^3-2)^8}\left(\frac{1}{2x} + 2 - \frac{24x^2}{x^3-2}\right).$$

bonus problem [5 points extra] Evaluate the limit $\lim_{x\to 0}\dfrac{e^{\sin x}-1}{x}$. Do not use L'Hospital's rule.

Solution: The limit represents the derivative of the function $f(x) = e^{\sin x}$ at 0.

$f'(x) = e^{\sin x}\cos x, \quad f'(0) = e^0 \cdot 1 = 1$.

Hence, $\lim_{x\to 0}\dfrac{e^{\sin x}-1}{x} = f'(0) = 1$.