11am **Quiz 7**

Fall 2012 **S o l u t i o n s**

Math 0220

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1. Find the limit. Use l'Hospital rule if appropriate.

(a) [2 points] $\lim_{t\to\infty} \frac{\ln\ln t}{t}$.

Solution: $\lim_{t\to\infty} \frac{\ln\ln t}{t} \left["\frac{\infty}{\infty}" \right] \stackrel{H}{=} \lim_{t\to\infty} \frac{(\ln t)^{-1}(t^{-1})}{1} = \lim_{t\to\infty} \frac{1}{t\ln t} = 0.$

(b) [3 points] $\lim_{x\to 0^+} (\sqrt{x})^x$.

Solution: $(\sqrt{x})^x = e^{\ln(\sqrt{x})^x} = e^{x/2\cdot\ln x}$

$\lim_{x\to 0^+} (\sqrt{x})^x = \lim_{x\to 0^+} e^{x/2\cdot\ln x} = e^{\lim_{x\to 0^+} x/2\cdot\ln x}$

$\lim_{x\to 0^+} x/2\cdot\ln x = \lim_{x\to 0^+} \frac{\ln x}{2\,x^{-1}} \left["\frac{\infty}{\infty}" \right] \stackrel{H}{=} \lim_{x\to 0^+} \frac{x^{-1}}{-2x^{-2}} = \lim_{x\to 0^+} \frac{x}{-2} \stackrel{DSP}{=} 0.$

Hence, $\lim_{x\to 0^+} (\sqrt{x})^x = e^0 = 1$.

2. [5 points] Find the local maximum and minimum values of the function $f(x) = 2x^{3/2} - 6x$.

Solution: Domain of the function is $x \geq 0$.

CNs: $f'(x) = 3\sqrt{x} - 6 = 0 \Rightarrow x = 4$. $f'(x)$ exists everywhere in the domain. The only CN is 4.

$f'(x) > 0 \Leftrightarrow x^{1/2} - 2 > 0 \Leftrightarrow x > 4$. Similarly $f'(x) < 0 \Leftrightarrow x < 4$.

It means that $f'(x)$ changes its sign from "-" to "+" at 4. Hence $f(x)$ has a local minimum at 4. The local minimum value of f is $f(4) = 2\cdot 8 - 24 = -8$.

bonus problem [5 points extra] Show that the equation $3x - 1 - \sin x = 0$ has exactly one real root.

Solution: Consider the function $f(x) = 3x - 1 - \sin x$. $f(0) = -1 < 0$, $f(1) = 2 - \sin 2 > 0$. On the interval $[0, 1]$ the function is continuous and satisties the inequality $f(0) < 0 < f(1)$. Applying the IVT with $N = 0$ we get that there is c in $(0, 1)$ such that $f(c) = 0$, which means that c is a real root of the given equation.

Assume that the equation and hence the function has two or more roots. Let a and b be two roots. Then $f(a) = f(b) = 0$. The function is continuous and differentiable everywhere on $(-\infty, \infty)$. By the MVT there is r such that $f'(c) = 0$. But $f'(x) = 3 - \cos x > 0$ for all x. It contradicts to the fact that at c the derivative is zero. Hence the assumption about the existence of at least two roots was wrong. Therefore, the function and hence the equation has exactly one root.