12pm **Quiz 7**

Fall 2012 **S o l u t i o n s**

Math 0220

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1. Find the limit. Use l'Hospital rule if appropriate.

(a) [2 points] $\lim\limits_{t\to 1} \dfrac{\ln t}{\sin \pi t}$.

Solution: $$\lim_{t\to 1} \frac{\ln t}{\sin \pi t} \quad \left[\text{"}\tfrac{0}{0}\text{"}\right] \stackrel{H}{=} \lim_{t\to 1} \frac{1/t}{\pi \cos \pi t} \stackrel{DSP}{=} \frac{1}{\pi(-1)} = -\frac{1}{\pi}.$$

(b) [3 points] $\lim\limits_{x\to\infty} x \tan(1/x)$.

Solution: $$\lim_{x\to\infty} x\tan(1/x) \quad [\text{"}0\cdot\infty\text{"}] = \lim_{x\to\infty} \frac{\tan(x^{-1})}{x^{-1}} \quad \left[\text{"}\tfrac{0}{0}\text{"}\right] \stackrel{H}{=} \lim_{x\to\infty} \frac{\sec^2(x^{-1})(-x^{-2})}{-x^{-2}}$$
$$= \lim_{x\to\infty} \sec^2(x^{-1}) = \sec^2 0 = 1.$$

2. [5 points] Verify that the function $f(x) = \dfrac{x}{x+1}$ satisfies the hypotheses of the MVT on the interval $[0,3]$. Then find all numbers c that satisfy the conclusion of the MVT.

Solution: The only discontinuity that $f(x)$ has is at -1. Hence $f(x)$ is continuous on $[0,3]$. It is a rational function and hence differentiable on $(0,3)$. Therefore, $f(x)$ satisfies the hypotheses of the MVT on the interval $[0,3]$.

By the MVT there is a number c such that $f'(c) = \dfrac{f(3)-f(0)}{3-0} = \dfrac{3/4-0}{3} = \dfrac{1}{4}$.

$$f'(x) = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}.$$

Therefore, to find c we need to solve the equation $\dfrac{1}{(c+1)^2} = \dfrac{1}{4} \Leftrightarrow (c+1)^2 = 4$.

Then $c+1=2$ or $c+1=-2$. Two solutions $c=1$ and $c=-3$. The last is not in $(0,3)$.

Hence, $c=1$.

bonus problem [5 points extra] Evaluate the limit $\lim_{x\to 0} 3(\sin x)^{\tan x}$.

Solution: $(\sin x)^{\tan x} = e^{\ln(\sin x)^{\tan x}} = e^{\tan x \cdot \ln(\sin x)}$

$$\lim_{x\to 0} 3(\sin x)^{\tan x} = 3 \lim_{x\to 0} (\sin x)^{\tan x} = 3 \lim_{x\to 0} e^{\tan x \cdot \ln(\sin x)} = 3e^{\lim_{x\to 0} \tan x \cdot \ln(\sin x)}$$

$$\lim_{x\to 0} \tan x \cdot \ln(\sin x) \; [\text{"}0 \cdot \infty\text{"}] = \lim_{x\to 0} \frac{\ln(\sin x)}{\cot x} \; \left[\text{"}\frac{\infty}{\infty}\text{"}\right] \stackrel{H}{=} \lim_{x\to 0} \frac{\cot x}{-\csc^2 x} = -\lim_{x\to 0} \cos x \sin x \stackrel{DSP}{=} 0.$$

Hence, $\lim_{x\to 0} 3(\sin x)^{\tan x} = 3e^0 = 3$.