11am

Quiz 8

Fall 2012

S o l u t i o n s

Math 0220

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1. [5 points] Evaluate the definite integral $\int\limits_0^4 \sqrt{16-x^2}\, dx$.

Solution: The integral represents the area of a quarter of the circle of radius 4 in the first quadrant, centered at the origin.

The area is $\dfrac{\pi \cdot 4^2}{4} = 4\pi$.

Hence, $\int\limits_0^4 \sqrt{16-x^2}\, dx = 4\pi$.

2. [5 points] Find the average value f_{ave} of the function $f(x) = 3x^2 - 2x$ on the interval $[1, 3]$.

Solution: $f_{ave} = \dfrac{1}{3-1} \int\limits_1^3 (3x^2 - 2x)\, dx = \dfrac{1}{2}\left[x^3 - x^2\right]_1^3 = \dfrac{1}{2}\left[(27-9) - (1-1)\right] = 18/2 = 9.$

bonus problem [5 points extra] Evaluate the definite integral $\int\limits_0^2 \dfrac{x^3}{2}\, dx$ as a limit of its Riemann's sum. No credit be given if no Riemann's sum is used.

Solution: $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$. The right end point of i^{th} interval is $x_i = 0 + i\Delta x = \dfrac{2i}{n}$.

Then $\int\limits_0^2 \dfrac{x^3}{2}\, dx = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{2} \cdot \left(\dfrac{2i}{n}\right)^3 \cdot \dfrac{2}{n} = 8 \lim\limits_{n\to\infty} \dfrac{1}{n^4} \sum\limits_{i=1}^{n} i^3 = 8 \lim\limits_{n\to\infty} \dfrac{1}{n^4} \left(\dfrac{n(n+1)}{2}\right)^2$

$= 2 \lim\limits_{n\to\infty} \dfrac{n^2(n+1)^2}{n^4} = 2.$