12pm

Quiz 8

Fall 2012

S o l u t i o n s

Math 0220

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1. [5 points] Find an expression of the area under the graph of the function

$f(x) = \dfrac{\ln x}{x^2}$, $3 \le x \le 7$ as a limit of its Riemann's sum. Use the right end points.

Do not evaluate the limit.

Solution: $\Delta x = \dfrac{7-3}{n} = \dfrac{4}{n}$, $x_i = 3 + i\Delta x = 3 + \dfrac{4i}{n}$.

The area is $\lim\limits_{n\to\infty} \dfrac{4}{n} \sum\limits_{i=1}^{n} \dfrac{\ln(3+\frac{4i}{n})}{(3+\frac{4i}{n})^2}$

2. [5 points] Evaluate the definite integral $\int\limits_1^2 \dfrac{x+5x^7}{x^3}\,dx$.

Solution: $\int\limits_1^2 \dfrac{x+4x^7}{x^3}\,dx = \int\limits_1^2 (x^{-2}+5x^4)\,dx = \left[-x^{-1}+x^5\right]_1^2 = (-1/2+32)-(-1+1) = 31\dfrac{1}{2}$.

bonus problem [5 points extra] A stone was dropped off a cliff and hit the ground with a speed of 144 ft/s. What is the height of the cliff?

Solution: The motion is vertical and we choose the positive direction to be upward with the origin at the bottom of the cliff. Then the acceleration is $a(t) = -32$ ft/s^2. Taking antiderivative, we get $v(t) = -32t + v_0$. Since $v(0) = 0$ we get $v_0 = 0$ and $v(t) = -32t$. Then, taking antiderivative again, we get $s(t) = -16t^2 + s_0$. To find s_0 we need to know the time when the stone hits the ground. Using $v(t) = -32t = -144$ we get $t = \dfrac{9}{2}$ s. Then $s(9/2) = 0$. On the other hand $s(9/2) = -16 \cdot \left(\dfrac{9}{2}\right)^2 + s_0$. Hence, $-16 \cdot \left(\dfrac{9}{2}\right)^2 + s_0 = 0$ or $s_0 = 324$ ft.

The height of the cliff is 324 ft.