11am **Quiz 9**

Fall 2012 **S o l u t i o n s**

Math 0220

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1. [5 points] Evaluate the definite integral $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$.

Solution: Substitution: $u = \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}\, dx$, $dx = 2\sqrt{x}\, du$, $u(1) = 1$, $u(4) = 2$.

Hence, $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = \int_1^2 \frac{e^u}{\sqrt{x}} \cdot 2\sqrt{x}\, du = 2\int_1^2 e^u\, du = 2\ e^u\Big|_1^2 = 2\left(e^2 - e\right).$

2. [5 points] Evaluate the definite integral $\displaystyle\int_1^2 \frac{\ln x}{x^2}\, dx$.

Solution: $\displaystyle\int_1^2 \frac{\ln x}{x^2}\, dx = \int_1^2 x^{-2} \ln x\, dx$

By parts: $u = \ln x$, $du = \dfrac{1}{x}\, dx$, $dv = x^{-2}\, dx$, $v = -x^{-1} = -\dfrac{1}{x}$.

Then $\displaystyle\int_1^2 \frac{\ln x}{x^2}\, dx = -\frac{\ln x}{x}\Big|_1^2 - \int_1^2 \left(-\frac{1}{x}\right) \cdot \frac{1}{x}\, dx = -\frac{\ln 2}{2} + \int_1^2 x^{-2}\, dx$

$\displaystyle = -\frac{\ln 2}{2} + \left[-\frac{1}{x}\right]_1^2 = -\frac{\ln 2}{2} + \frac{1}{2} = \frac{1 - \ln 2}{2}.$

bonus problem [5 points extra] Evaluate the integral $\displaystyle\int x\sqrt{x^2+1}\, dx$

Solution: Trig. substitution: $x = \tan\theta$, $dx = \sec^2\theta\, d\theta$,

$\sqrt{x^2+1} = \sqrt{\tan^2\theta +} = \sqrt{\tan^2\theta + 1} = \sec\theta.$

Then $\displaystyle\int x\sqrt{x^2+1}\, dx = \int \tan\theta \cdot \sec\theta \cdot \sec^2\theta\, d\theta = \int \sec^2\theta \cdot \tan\theta \sec\theta\, d\theta$

[substitution: $u = \sec\theta$, $du = \tan\theta \sec\theta\, d\theta$]

$$= \int u^2 \, du = \frac{u^3}{3} + C = \frac{\sec^3 \theta}{3} + C = \frac{\left(\sqrt{x^2+4}\right)^3}{3} + C = \frac{(x^2+4)^{3/2}}{3} + C$$

Note: $\sec \theta = \sqrt{x^2 + 1}$