12pm **Quiz 9**

Fall 2012 **S o l u t i o n s**

Math 0220

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1. [5 points] Find $g'(x)$ if $g(x) = \int_{x^2}^{3} \frac{\sqrt{\sin t}}{t + \cos t}\, dt$.

Solution: By FTC, part 1: $g'(x) = 0 - \frac{\sqrt{\sin(x^2)}}{x^2 + \cos(x^2)} \cdot (x^2)' = -\frac{2x\sqrt{\sin(x^2)}}{x^2 + \cos(x^2)}$.

2. [5 points] Evaluate the definite integral $\int_0^1 \frac{x}{e^{2x}}\, dx$.

Solution: $\int_0^1 \frac{x}{e^{2x}}\, dx = \int_0^1 xe^{-2x}\, dx$

By parts: $u = x$, $du = dx$, $dv = e^{-2x}\, dx$, $v = -\frac{e^{-2x}}{2}$.

Then $\int_0^1 \frac{x}{e^{2x}}\, dx = -\frac{xe^{-2x}}{2}\Big|_0^1 - \int_1^2 \left(-\frac{e^{-2x}}{2}\right) dx = -\frac{e^{-2}}{2} + \int_0^1 \frac{e^{-2x}}{2}\, dx$

$= -\frac{e^{-2}}{2} + \left[-\frac{e^{-2x}}{4}\right]_0^1 = -\frac{e^{-2}}{2} - \frac{e^{-2}}{4} + \frac{1}{4} = -\frac{3e^{-2}}{4} + \frac{1}{4} = \frac{1}{4} - \frac{3}{4e^2} = \frac{e^2 - 3}{4e^2}$.

bonus problem [5 points extra] Evaluate the integral $\int \tan^3 x \sec^4 x\, dx$

Solution: $\int \tan^3 x \sec^4 x\, dx = \int \tan^3 x \sec^2 x \sec^2 x\, dx = \int \tan^3 x \left(1 + \tan^2 x\right) \sec^2 x\, dx$.

[substitution: $u = \tan x$, $du = \sec^2 x\, dx$, $dx = \frac{du}{\sec^2 x}$]

$= \int u^3 \left(1 + u^2\right) \sec^2 x \frac{du}{\sec^2 x} = \int \left(u^3 + u^5\right) du = \frac{u^4}{4} + \frac{u^6}{6} + C = \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C$.