1-1:50pm

Midterm Exam 1

Spring 2012 Math 0220

100 points total **Your name:** _____

No calculators. Show all your work (no work = no credit). Explain every step. Write neatly.

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1. (10 points) Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x \to -5^-} \frac{3x + 15}{|x + 5|}$$

In your work mention what Rules, Laws, Theorems or Formulas you use.

2. (a) (10 points) Find the functions $f \circ g$ and $g \circ f$ if

$$f(x) = 2 - \frac{1}{x} \quad \text{and} \quad g(x) = \frac{x-1}{x-2}$$

Simplify your answers.

(b) (5 points) Find the domain of the function $f \circ g$.

3. (15 points) Use the Intermediate Value Theorem to show that there is root of the equation $x^2 - 1 = \sin x$ in the interval $(0, \pi)$.

4. (10 points) Find all horizontal and vertical asymptotes of the curve

$$y = \frac{\sqrt{4x^2 + 5}}{4x + 5}$$

Justify your answer by calculating corresponding limits.

5. For the function $f(x) = 2\cos x - \sqrt{3x}$

(a) (5 points) find its first derivative $f'(x)$.

(b) (5 points) find its second derivative $f''(x)$.

6. (10 points) Find an equation of the tangent line to the curve

$$y = (2 + 3x)\cos x$$

at the point which x-coordinate is 0.

7. (15 points) Using any method find the derivative $G'(\theta)$ of the function

$$G(\theta) = \sqrt{\sin(\cos^2\theta)}$$

In your work mention what Rules, Laws, Theorems or Formulas you use.

8. (15 points) Use implicit differentiation to find an equation of the tangent line to the curve $2\cos x \sin y = 1$ at the point $(\pi/4, \pi/4)$.

bonus problem [8 points extra] Consider the functions

$$f(x) = \begin{cases} -x^2 & \text{if} \quad x < 0 \\ \\ x + 1 & \text{if} \quad x \geq 0 \end{cases} \qquad \text{and} \quad g(x) = b^2x^2 + bx + b^2$$

Find all values of b such that $g(f(x))$ is continuous everywhere.