9-9:50am

Midterm Exam 1

Spring 2012 Math 0220

100 points total **Your name:** _____

No calculators. Show all your work (no work = no credit). Explain every step. Write neatly.

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1. (10 points) Find the domain of the function

$$g(t) = \frac{\sqrt{1-t^2}}{2\sin t - 1}$$

2. (15 points) Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x \to 0} x^3 \sin\left(\frac{3}{x}\right)$$

In your work mention what Rules, Laws, Theorems or Formulas you use.

3. (10 points) Define if the function

$$f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if} \quad x < 2 \\ 2x - 1 & \text{if} \quad x \geq 2 \end{cases}$$

is continuous or discontinuous at the point $a = 2$. Support your answer by using the definition of continuity. No credit will be given if you do not use the definition in your proof. Do not draw a graph of the function.

4. (10 points) Sketch the graph of an example of a function $g(x)$ if it satisfies all the given conditions

$$g(0) = 2, \quad g'(0) = -1, \quad g(2) = -1, \quad g'(2) = -\frac{1}{2}, \text{ and } g'(5) = 2$$

Mark all the essential points on the axes.

5. A particle moves according to a law of motion $s(t) = t^3 - 12t^2 + 36t$, $t \geq 0$, where t is measured in seconds and s in feet.

   (a) (5 points) What is its velocity after 3 seconds?

   (b) (5 points) When is the particle at rest?

   (c) (5 points) Find the total distance traveled during the first 8 seconds.

6. (10 points) Using any method find the derivative $f'(x)$ of the function

$$f(x) = \frac{\sqrt{2x} + 2}{\sqrt{2x} - 2}$$

Simplify your answer. In your work mention what Rules, Laws, Theorems or Formulas you use.

7. (15 points) Find the first and second derivatives of the function

$$g(x) = \sqrt{2x^2 - 1}$$

Simplify your answer. In your work mention what Rules, Laws, Theorems or Formulas you use.

8. (15 points) Use implicit differentiation to find an equation of the tangent line to the curve $x^{2/3} + y^{2/3} = 5$ at the point $(8, 1)$.

bonus problem [8 points extra] Use the definition of the derivative to find the derivative of the function $f(x) = g(cx)$, where $g(x)$ is a differentiable function and c is a constant. No credit be given if the derivative will be found without using the definition.