Midterm Exam 2

100 points total   **Your name:** _____

No calculators. Show all your work (no work = no credit). Explain every step. Write neatly.

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1. (15 points) Find $\left(f^{-1}\right)'(2)$ if $f(x) = \dfrac{1}{x-1}, \ x > 1.$

2. (15 points) A piece of cloth is recovered by some archaeologists. Upon examination, it is found that 75% of the original Carbon 14 remains in the cloth. If the half-life of Carbon 14 is 5800 years, calculate the age of the piece of cloth.

3. (15 points) Find the limit

$$\lim_{x \to \pi} \frac{\cos^3(x/2)}{\sin x}$$

4. (15 points) Find the absolute maximum and absolute minimum values of the function

$$f(x) = xe^{-x^2/2} \quad \text{on the interval} \quad [-1, 4]$$

5. (15 points) Find the dimensions of a rectangle with area 400 m^2 whose perimeter as small as possible. Support your answer.

6. (10 points) Find the limit

$$\lim_{x \to \infty} \left(2^{-x} \sin 3x \right)$$

7. (15 points) For the equation $x^2 = 2$ use Newton's method with the initial approximation $x_1 = 2$ to find the third approximation x_3 to the positive root.

bonus problem [10 points extra] Show that the equation $2\cos x - 3x = 0$ has exactly one real root.