9-9:50am

Midterm Exam 2

Spring 2012 Math 0220

100 points total

Your name:

No calculators. Show all your work (no work = no credit). Explain every step. Write neatly.

1. (10 points) Find a formula for the inverse of the function

$$f(x) = \frac{1 - e^x}{1 + e^x}$$
 when $-1 < x < 1$

2. (15 points) Suppose that the population of a colony of bacteria increases exponentially. At the start of an experiment, there are 6,000 bacteria, and one hour later, the population has increased to 6,400. How long will it take for the population to reach 10,000?

3. (15 points) Find the derivative of the function and simplify it if possible

$$f(x) = x \cos^{-1}\left(e^{3x}\right)$$

4. (15 points) Find the limit

$$\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$$

- 5. For the function $f(x) = xe^x$
 - (a) (5 points) Find the intervals on which f is increasing or decreasing.
 - (b) (5 points) Find the local maximum and local minimum values of f.
 - (c) (5 points) Find the intervals of concavity (where f is CU and CD) and the inflection points.

6. (15 points) A box with a square base and open top must have a volume of $8000~\rm{cm^2}$. Find the dimensions of the box that minimize the amount of material used.

7. (15 points) For the equation $x^3 - 2x - 5 = 0$ use Newton's method with the initial approximation $x_1 = 2$ to find the second approximation x_2 to the positive root.

bonus problem [10 points extra] Find the exact value of the expression $2^{3\log_2 3} + \tan(\sin^{-1}\frac{1}{2})$.