10am

Quiz 1

Spring 2012 **Your name:** ______________________

Math 0220 **Your TA's name:** ______________________

No calculators. Show all your work (no work = no credit). Write neatly.

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1. [5 points] Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{h\to 0}\left(\frac{1}{h^2-h}+\frac{1}{h}\right)$$

In your work mention what Rules, Laws, or Formulas you use.

$$\lim_{h\to 0}\left(\frac{1}{h^2-h}-\frac{1}{h}\right)=\lim_{h\to 0}\left(\frac{1}{h(h-1)}+\frac{1}{h}\frac{(h-1)}{(h-1)}\right)$$

$$=\lim_{h\to 0}\frac{1+h-1}{h(h-1)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(h-1)}$$

$$=\lim_{h\to 0}\frac{1}{h-1}$$

$$(DSP)\rightarrow = \frac{1}{0-1}$$

$$-1$$

2. (a) [4 points] Express the functions $F(x) = \csc^5(\sqrt[3]{x})$ in the form $f \circ g \circ h$.

(b) [1 point] What is the domain of $h(x)$?

$$F(x) = \csc^5(\sqrt[3]{x}) = (\csc(\sqrt[3]{x}))^5$$

a) $h(x) = \sqrt[3]{x}$

$g(x) = \csc(x)$

$f(x) = x^5$

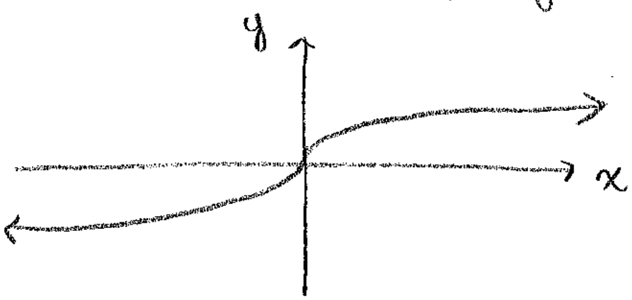
Always check! $\Rightarrow$ $f(g(h(x))) = f(g(\sqrt[3]{x}))$

$= f(\csc(\sqrt[3]{x}))$

$= \csc^5(\sqrt[3]{x})$

b) $h(x) = \sqrt[3]{x} = x^{1/3}$ $\longrightarrow$ domain of $h(x)$ is all real numbers, i.e. $\mathbb{R}$

You should know the shape of the graph of $x^{1/3}$:



3. [5 points] Evaluate the difference quotient $\frac{f(x)-f(1)}{x-1}$ for the function $f(x) = \frac{x+5}{x+1}$.

$$\frac{f(x)-f(1)}{x-1} = \frac{\frac{x+5}{x+1} - \frac{1+5}{1+1}}{x-1} = \frac{\frac{x+5}{x+1} - 3\frac{(x+1)}{(x+1)}}{x-1}$$

$$= \frac{\frac{x+5}{x+1} - \frac{3x+3}{x+1}}{x-1}$$

$$= \frac{\frac{x+5-3x-3}{x+1}}{x-1} \cdot \frac{1}{x-1}$$

$$= \frac{-2x+2}{x+1} \cdot \frac{1}{x-1}$$

$$= \frac{-2\cancel{(x-1)}}{x+1} \cdot \frac{1}{\cancel{x-1}}$$

$$= \frac{-2}{x+1}$$

bonus problem [5 points extra] Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}.$$

Problem: DS gives us $1-1=0$ in denominator.
First trick to try: rationalize numerator.

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{x-1}{(\sqrt[3]{x}-1)(\sqrt{x}+1)} = (*)$$

still a problem, we need to cancel this

Q: If we factor out $\sqrt[3]{x}-1$ from the numerator $x-1$, what is left? Can use long division to find out!

$$\begin{array}{r|l} & x^{2/3} + x^{1/3} + 1 \\ x^{1/3}-1 & x - 1 \\ & -(x - x^{2/3}) \\ & x^{2/3} - 1 \\ & -(x^{2/3} - x^{1/3}) \\ & x^{1/3} - 1 \\ & (x^{1/3}-1) \\ & 0 \end{array}$$

So, from (*)

$$(*) = \lim_{x\to 1} \frac{\cancel{(x^{1/3}-1)}(x^{2/3}+x^{1/3}+1)}{\cancel{(x^{1/3}-1)}(\sqrt{x}+1)} = \lim_{x\to 1} \frac{x^{2/3}+x^{1/3}+1}{\sqrt{x}+1}$$

$$\overset{\text{DSP + Limit Laws}}{=} \frac{1+1+1}{1+1} = \frac{3}{2}$$