9am

Quiz 1

Spring 2012 **Your name:** ______________________

Math 0220 **Your TA's name:** ______________________

No calculators. Show all your work (no work = no credit). Write neatly.

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1. (a) [3 points] Find the functions $f \circ g$, $g \circ f$ if

$$f(x) = \cos x, \quad g(x) = \sqrt{\frac{1}{4} - x^2}$$

(b) [1 point] Find the domain (maximal possible) of the function $f \circ g$.

(c) [1 point] Find the domain of $g \circ f$ inside the interval $[-\pi, \pi]$.

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{1/4 - x^2}) = \cos(\sqrt{1/4 - x^2})$

$(g \circ f)(x) = g(f(x)) = g(\cos(x)) = \sqrt{1/4 - \cos^2 x}$

b) Domain of $f \circ g$ is $D_{f \circ g} = [-1/2, 1/2]$

• must have $1/4 - x^2 \geq 0 \Leftrightarrow (1/2 - x)(1/2 + x) \geq 0$

$\Leftrightarrow |x| \leq 1/2$ $\quad \ominus \; -1/2 \; \oplus \; 1/2 \; \ominus$

(*) • domain of cosine is all real numbers, so no restrictions there

c) Domain of $g \circ f$ is $D_{g \circ f} = [-2\pi/3, -\pi/3] \cup [\pi/3, 2\pi/3]$

• (*)

• must have $1/4 - \cos^2 x \geq 0$, i.e. $|\cos x| \leq 1/2$ or $-1/2 \leq \cos x \leq 1/2$

$x \in [-2\pi/3, -\pi/3] \cup [\pi/3, 2\pi/3]$

See last page for details

2. [5 points] Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x\to 0} \frac{\sin^2(2x)}{4x^2}$$

In your work mention what Rules, Laws, or Formulas you use.

$$\lim_{x\to 0} \frac{\sin^2(2x)}{4x^2} = \lim_{x\to 0} \frac{(\sin(2x))^2}{(2x)^2} = \lim_{x\to 0,} \left(\frac{\sin(2x)}{2x}\right)^2$$

Limit Law (Product/ Mult.) $= \left(\lim_{x\to 0} \frac{\sin(2x)}{2x}\right)\left(\lim_{x\to 0} \frac{\sin(2x)}{2x}\right)$

Property (*) $= 1 \cdot 1$

$= 1$

((*) It is known that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$)

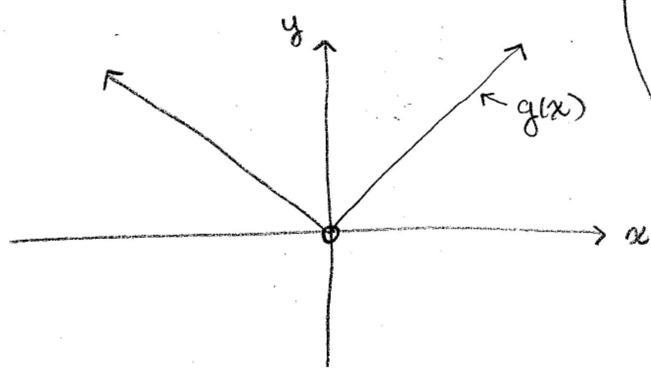
3. [5 points] Find the $\overbrace{\text{domain}}^{(1)}$ and $\overbrace{\text{sketch the graph}}^{(4)}$ of the function $g(x) = \dfrac{x^2}{|x|}$.

Domain: $\mathbb{R}\setminus\{0\}$ or $(-\infty,0)\cup(0,\infty)$ since can't have zero in denominator

Recall $\quad |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Thus, $\quad g(x) = \left.\begin{cases} \dfrac{x^2}{x} & x > 0 \\ \dfrac{x^2}{-x} & x < 0 \end{cases}\right\} = \left\{\begin{matrix} x & x > 0 \\ -x & x < 0 \end{matrix}\right\} = |x|, \text{ for } x \neq 0$

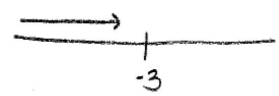
2 points → graph
2 points for



* It is not acceptable to graph by plotting points. You must graph algebraically, demonstrating a basic understanding of piecewise functions, lines, parabolas, trig functions, etc. & transformations of these functions, i.e. the entire purpose of section 1.2!

bonus problem [5 points extra] Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x\to -3^-} \left(\frac{7x+21}{2|x+3|} - x - 3 \right).$$

Note: $x \to -3^-$ means $x < -3$ 

Thus $x+3 < -3+3 = 0$, so

$|x+3| = -(x+3)$

$$\lim_{x\to -3^-} \left(\frac{7x+21}{2|x+3|} - x - 3 \right) = \lim_{x\to -3^-} \left(\frac{7x+21}{-2(x+3)} - x - 3 \right)$$

$$= \lim_{x\to -3^-} \left(\frac{7\cancel{(x+3)}}{-2\cancel{(x+3)}} - x - 3 \right)$$

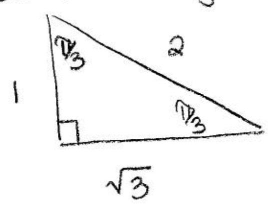
$$= \lim_{x\to -3^-} \left(-\tfrac{7}{2} - x - 3 \right)$$

(DSP) $= -\tfrac{7}{2} - (-3) - 3$

$= -\tfrac{7}{2}$

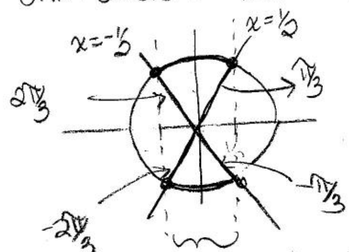
1. c) We need to find where $|\cos x| \le \frac{1}{2}$

30-60-90 Triangle



$\cos \frac{\pi}{3} = \frac{1}{2}$

Unit Circle: sine = height cosine = horizontal position



So $\cos \frac{\pi}{3} = \cos -\frac{\pi}{3} = \frac{1}{2}$

cosine is even

$\cos \frac{2\pi}{3} = \cos -\frac{2\pi}{3} = -\frac{1}{2}$

need angles inside of here

Hence $|\cos x| \le \frac{1}{2}$ when $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$ or $-\frac{2\pi}{3} \le x \le -\frac{\pi}{3}$

Or if we look at the graph of cosine