10am

Quiz 2

Spring 2012 **Your name:** ______________________

Math 0220 **Your TA's name:** ______________________

No calculators. Show all your work (no work = no credit). Write neatly.

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1. [5 points] Evaluate the limit, if it exists. If it does not exist explain why.

$$\lim_{x\to\infty} \cos 2x$$

The limit does not exist since as x increases, $2x$ increases & the values of $\cos 2x$ oscillate between 1 & -1 infinitely often (cosine is periodic).

Similar to: Example 8 section 1.6

2. [5 points] Is the function

$$h(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if} \quad x \neq 1 \\ 1 & \text{if} \quad x = 1 \end{cases}$$

continuous or discontinuous at $a = 1$? Show why it is continuous (or discontinuous) at this point.

h(x) is continuous at a=1 if 1 is in the domain of h &

1) $\lim_{x \to 1} h(x)$ exists

2) $\lim_{x \to 1} h(x) = h(1)$

We check (1)

$$\lim_{x \to 1} h(x) = \lim_{x \to 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \to 1} \frac{x\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \lim_{x \to 1} \frac{x}{x+1} \overset{\text{(DSP)}}{=} \frac{1}{1+1} = \frac{1}{2}$$

We check (2)

$h(1) = 1$, but $\lim_{x \to 1} h(x) = \frac{1}{2}$

Since (2) fails, h(x) is discontinuous at $x = 1$

Similar to Example 2 section 1.5

Homework 1.5.29, 1.5.33

3. [5 points] Using the definition of the derivative find $f'(a)$ if $f(x) = x^2 - 5x$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{[(a+h)^2 - 5(a+h)] - [a^2 - 5a]}{h}$$

$$= \lim_{h\to 0} \frac{[a^2 + 2ah + h^2 - 5a - 5h] - a^2 + 5a}{h}$$

$$= \lim_{h\to 0} \frac{2ah - h^2 - 5h}{h} \underset{(DSP)}{=} \lim_{h\to 0} \frac{\cancel{h}(2a - h - 5)}{\cancel{h}}$$

$$= \lim_{h\to 0} 2a - h - 5 = 2a - 0 - 5 = 2a - 5$$

OR

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a} = \lim_{x\to a} \frac{(x^2 - 5x) - (a^2 - 5a)}{x - a}$$

$$= \lim_{x\to a} \frac{x^2 - 5x - a^2 + 5a}{x-a} = \lim_{x\to a} \frac{(x^2 - a^2) - 5(x-a)}{x-a}$$

$$= \lim_{x\to a} \frac{(x+a)(x-a) - 5(x-a)}{x-a} = \lim_{x\to a} \frac{\cancel{(x-a)}[(x+a)-5]}{\cancel{x-a}}$$

$$= \lim_{x\to a} x + a - 5 \underset{(DSP)}{=} a + a - 5 = 2a - 5$$

Similar to Example 4 section 2.1

Homework 2.1.26

bonus problem [5 points extra] Show that any line of nonzero slope that passes through the point $(0, 1)$ must intersect the graph of $\sin x$ at least once.

General Equation of a line w/ nonzero slope, m, that passes through $(0,1)$ is

$$y-1=m(x-0)$$
$$y=mx+1$$

We are asked to show the line & $\sin x$ intersect at least once, i.e.

$$mx+1=\sin x \quad \text{for some real } x$$

This is an IVT problem!

1) Define $f(x)=mx+1-\sin x$. Then f is continuous as a difference of 2 continuous functions, $\sin x$, & $mx+1$.

2) We find $a<b$ such that $f(a)<0<f(b)$ or $f(b)<0<f(a)$

<u>Case (i)</u> $m>0$

<u>Let</u> k be a positive integer such that $m>\frac{1}{k\pi} \Leftrightarrow -km\pi<-1$

$km\pi>1$

$f(-k\pi)=-km\pi+1<0$

$f(k\pi)=km\pi+1>0$

Then $f(-k\pi)<0<f(k\pi) \longrightarrow (f(a)<0<f(b))$

<u>Case (ii)</u> $m<0$

$km\pi<-1$

Let k be a positive integer such that $m<\frac{-1}{k\pi} \Leftrightarrow -km\pi>1$

$f(-k\pi)=-km\pi+1>0$

$f(k\pi)=km\pi+1<0$

Then $f(k\pi)<0<f(-k\pi) \longrightarrow (f(b)<0<f(a))$

3) In either case, by IVT, there is $c\in(-k\pi,k\pi)$ such that $f(c)=0$, i.e. $mc+1=\sin c$ (the graphs intersect)