9-9:50am

Midterm Exam 1

Spring 2013 Math 0220

100 points total **Your name:** _____

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. [10 points] Calculate $y'(0)$ if $\sin(xy) = (x+1)^{3/2} - y$.

2. Find the limit, probably infinite, if it exists. If the limit does not exist explain why. If it is infinity, then find its sign. Provide all the necessary steps or explanations. You can use any known method.

(a) [10 points] $L = \lim_{x \to -2} \dfrac{\sqrt{x+3} - 1}{x+2}$

(b) [10 points] $\lim_{x \to 0} \dfrac{x^2}{|x|}$.

(c) [10 points] $L = \lim_{h \to 0} \dfrac{f(5+h) - f(5)}{h}$, where $f(x) = x^3$.

3. [15 points] A screen saver displays the outline of a 3 cm by 2 cm rectangle and then expands the rectangle in such a way that the 2 cm side is expanding at the rate of 2 cm/sec and the proportions of the rectangle never change. How fast is the area of the rectangle increasing when its dimensions are 12 cm by 8 cm?

4. [10 points] Find an equation of the normal line to the curve $y = \dfrac{\cos^2 x}{2}$ at the point $\left(\frac{\pi}{4}, \frac{1}{4}\right)$. Write the answer in the slope-intercept form.

5. Evaluate derivatives. Mention rules used.

(a) [10 points] Evaluate $f'(\pi)$ if $f(x) = \dfrac{x}{\cos x}$.

(b) [10 points] Evaluate $f''(3)$ if $f(x) = \sqrt{x^2 - 5}$.

6. [15 points] Explain why the function

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

is discontinuous at $x = 1$. Find a continuous function $g(x)$ such that $g(x) = f(x)$ when $x \neq 1$.

bonus problem. [10 points extra] Find $y''(2)$ if $x^3 + y^3 = 7$.