# Midterm Exam 2

100 points total                 **Your name:** _____

No calculators, no notes, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. Find the limits. Use l'Hospital rule if appropriate. In your solution mention types of all indeterminate forms.

(a) [10 points]     $\displaystyle\lim_{x\to 0}\frac{x+\tan x}{\sin x}$.

(b) [10 points]     $\displaystyle\lim_{x\to -\infty} x^2 e^x$.

2. (a) [10 points] Find the linear approximation of the function $f(x) = x^{2/3}$ at $a = 64$.

(b) [10 points] Use the linearization above to approximate the number $63^{2/3}$.

3. [15 points] Find $(f^{-1})'(3)$ if $f(x) = x^4 + 3x^3 - 1$. [Hint: $f(1) = 3$].

4. [15 points] A house was purchased for $\$100,000$ in 2002. Six years after its value was $\$120,000$. Find the value of the house in 2014 (exactly twelve years after the purchase) if it grows exponentially. The result is an integer number. Find this number. Do not leave fractions, logs or exponents in your answer.

5. [15 points] Use optimization method to find a point on the line $y = -x + 4$ that is closest to the point $(3, 4)$

6. [15 points] For the equation $x^2 = 12$ use Newton's method with the initial approximation $x_1 = 3$ to find the third approximation $x_3$ to the positive root. (Write your answer as a reduced fraction).

bonus problem. [15 points extra] Show that the equation $4x+1 = \sin x$ has exactly one real root.