Math 0220 **Quiz 2**

Spring 2013 **Solutions**

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1. [5 points] Evaluate the limit $L = \lim_{x \to 0} \dfrac{x}{\sqrt{16-x}-4}$, if it exists.

Solution: $$L = \lim_{x\to 0} \frac{x}{\sqrt{16-x}-4} \cdot \frac{\sqrt{16-x}+4}{\sqrt{16-x}+4} = \lim_{x\to 0} \frac{x(\sqrt{16-x}+4)}{16-x-16}$$

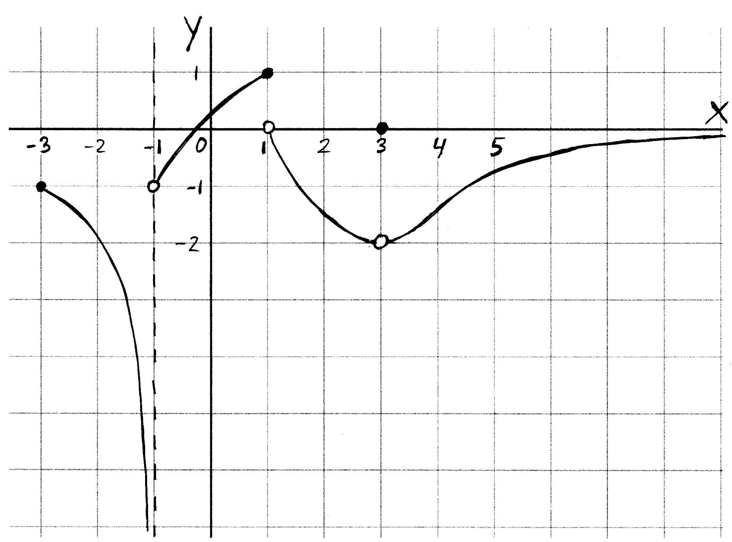
$$= \lim_{x\to 0} \frac{x(\sqrt{16-x}+4)}{-x} = \lim_{x\to 0} -(\sqrt{16-x}+4) = -(\sqrt{16}+4) = -8.$$

2. [5 points] Sketch the graph of an example of a function $g(x)$ if it has the domain $[-3,\infty)$ and satisfies all the given conditions. Mark all important points on the graph and the axes.

$g(-3) = -1$, $\lim_{x\to -1^-} g(x) = -\infty$, $\lim_{x\to -1^+} g(x) = -1$, $g(-1)$ is undefined,

$g(1) = 1$, $\lim_{x\to 1^-} g(x) = 1$, $\lim_{x\to 1^+} g(x) = 0$, $\lim_{x\to 3} g(x) = -2$, $g(3) = 0$, $\lim_{x\to\infty} g(x) = 0$.

Solution: One of the possible graphs:



Note: there is vertical asymptote $x = -1$ and horizontal asymptote $y = 0$.

bonus problem [5 points extra] Evaluate the difference quotient $\dfrac{f(2+h)-f(2)}{h}$ for the function $f(x) = \dfrac{x}{x-1}$.

Solution: $$\frac{f(2+h)-f(2)}{h} = \frac{\dfrac{2+h}{2+h-1} - 2}{h} = \frac{\dfrac{2+h}{1+h} - 2}{h} = \frac{2+h-2-2h}{(1+h)h} = \frac{-h}{(1+h)h}$$

$$= -\frac{1}{1+h}$$