Math 0220 **Quiz 3**

Spring 2013 **S o l u t i o n s**

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1. [5 points] Use implicit differentiation to find an equation of the tangent line to the curve $x^{2/3} + y^{2/3} = 5$ at the point $(8, 1)$.

Solution: Differentiate the equation with respect to x to find y':

$$\frac{d}{dx}\left[x^{2/3} + y^{2/3} = 5\right], \quad \tfrac{2}{3}\, x^{-1/3} + \tfrac{2}{3}\, y^{-1/3}\, y' = 0, \quad x^{-1/3} + y^{-1/3}\, y' = 0.$$

Now plug in 8 for x and 1 for y to obtain $\frac{1}{2} + y'(1) = 0$. Hence, the slope is $m = y'(1) = -\frac{1}{2}$.

Then the equation of the tangent line is $y = -\frac{1}{2}(x - 8) + 1$ or $y = -\frac{1}{2}\, x + 5$.

2. A particle moves according to a law of motion $s(t) = t^3 + 3t^2 - 24t, \quad t \geq 0,$ where t is measured in seconds and s in feet.

(a) (2 points) What is its velocity after 3 seconds?

(b) (1 points) When is the particle at rest?

(c) (2 points) Find the total distance traveled during the first 4 seconds.

Solution: (a) $v(t) = s'(t) = 3t^2 + 6t - 24 = 3(t^2 + 2t - 8) = 3(t - 2)(t + 4)$,

$v(3) = 3 \cdot 1 \cdot 7 = 21$ ft/sec.

(b) $3(t - 2)(t + 4) = 0, \;\; t_1 = 2$ sec, $\; t_2 = -4$ sec. The last root is not in the domain. Hence, the particle is at rest when $t = 2$ sec.

(c) Turning point is at $t = 2$ sec.

$s(t) = t(t^2 + 3t - 24), \quad s(0) = 0, \quad s(2) = 2(4 + 6 - 24) = -28, \quad s(4) = 4(16 + 12 - 24) = 16.$

The total distance traveled is $d = |s(2) - s(0)| + |s(4) - s(2)| = 28 + 44 = 72$ feet.

bonus problem [5 points extra] Use the definition of the derivative to compute the derivative of the function $g(x) = f(3x)$ at a number a, where f is a differentiable function. Write the answer in terms of the derivative of f.

Solution: $g'(a) = \lim\limits_{h \to 0} \dfrac{g(a + h) - g(a)}{h} = \lim\limits_{h \to 0} \dfrac{f(3(a + h)) - f(3a)}{h}$

$$= \lim_{h \to 0} \frac{3(f(3a+3h) - f(3a))}{3h}.$$

Now, put $u = 3h$. Then $u \to 0$ whenever $h \to 0$ and

$$g'(a) = 3 \lim_{u \to 0} \frac{f(3a+u) - f(3a)}{u} = 3f'(3a).$$