Math 0220 **Quiz 4**

Spring 2013 **S o l u t i o n s**

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1. [5 points] Use differentials to estimate the amount of paint needed to apply a coat of paint 0.03 cm thick to a hemispherical dome of the radius 20m.

Solution: The volume of a semisphere is $V = \frac{2}{3}\pi r^3$. The amount of paint needed is $\Delta V \approx dV = \frac{dV}{dr}dr = 2\pi r^2 dr \approx 2\pi r^2 \Delta r$.

It is given that $r = 20$ m and $\Delta r = 0.03$ cm $= 3 \cdot 10^{-2}$ cm $= 3 \cdot 10^{-2} \cdot 10^{-2}$ m.

Then $\Delta V \approx 2\pi \cdot 20^2 \cdot 10^{-2} \cdot 10^{-2} = 2\pi \cdot 4 \cdot 10^2 \cdot 10^{-2} \cdot 10^{-2} = 8\pi \cdot 10^{-2} = 0.08\pi$ m^3.

2. [5 points] A sample of tritium-3 decayed to 94.5% of its original amount after one year. What is the half-life of tritium-3? Leave ln in your answer.

Solution: The mass of tritium-3 is $m(t) = m_0 e^{kt}$. Then

$m(1) = m_0 e^k = 0.945 m_0 \quad \Rightarrow \quad e^k = 0.945 \quad \Rightarrow \quad m(t) = m_0 (0.945)^t$.

We need to find t such that $m(t) = \frac{1}{2}m_0$ or $m_0(0.945)^t = \frac{1}{2}m_0$ which gives $(0.945)^t = \frac{1}{2}$.

Applying ln to both sides we obtain $t \ln(0.945) = -\ln 2$.

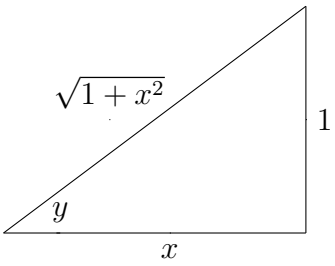
Then the half-live is $t = -\dfrac{\ln 2}{\ln 0.945}$ years.

[Note, that this number is positive b/c $\ln 0.945$ is negative].

Another way: We assume without loss of generality, that $m_0 = 1$ and use 2 as a base for the exponent. Then $m(t) = 2^{-kt}$. Let t be the half-life. Then $2^{-kt} = 1/2$ and $2^{-k} = 0.945 \quad \Rightarrow$ $0.945^t = 1/2 \quad \Rightarrow \quad t \ln 0.945 = \ln(1/2) = -\ln 2 \quad \Rightarrow \quad t = -\dfrac{\ln 2}{\ln 0.945}$ years.

bonus problem [5 points extra] Simplify the expression $\sin(\cot^{-1} x)$.

Solution: By the definition of an inverse function $y = \cot^{-1} x \quad \Leftrightarrow \quad x = \cot y$.



Or $\cot y = \dfrac{x}{1}$ (see the picture, y is an angle in the right triangle).

Then $\sin(\cot^{-1} x) = \sin y = \dfrac{1}{\sqrt{1+x^2}}$.