Math 0220 **Quiz 5**

Spring 2013 **S o l u t i o n s**

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1. Find the limit. Define undeterminate forms and apply l'Hospital's Rule where appropriate.

(a) [2 points] $L = \lim_{x\to 0} \dfrac{x}{(4-x)^{3/2}-8}$

Solution: $L = \lim_{x\to 0} \dfrac{x}{(4-x)^{3/2}-8} \left[\text{"}\frac{0}{0}\text{"}\right] \overset{LR}{=} \lim_{x\to 0} \dfrac{1}{\frac{3}{2}(4-x)^{1/2}\cdot(-1)}$

$$\overset{DSP}{=} \frac{1}{\frac{3}{2}\cdot 2\cdot(-1)} = -\frac{1}{3}.$$

(b) [3 points] $L = \lim_{x\to 0^+} (\sqrt{x})^x$.

Solution: $(\sqrt{x})^x = e^{\ln(\sqrt{x})^x} = e^{x/2\cdot\ln x}$

$$\lim_{x\to 0^+} (\sqrt{x})^x = \lim_{x\to 0^+} e^{x/2\cdot\ln x} = e^{\lim_{x\to 0^+} x/2\cdot\ln x}$$

$$\lim_{x\to 0^+} x/2\cdot\ln x = \lim_{x\to 0^+} \frac{\ln x}{2\,x^{-1}} \left[\text{"}\frac{\infty}{\infty}\text{"}\right] \overset{LR}{=} \lim_{x\to 0^+} \frac{x^{-1}}{-2x^{-2}} = \lim_{x\to 0^+} \frac{x}{-2} \overset{DSP}{=} 0.$$

Hence, $L = e^0 = 1$.

2. [5 points] Use guidelines of the section 4.4 to sketch the curve $y = x - 3x^{1/3}$.

Solution: We define $f(x) = x - 3x^{1/3}$.

A. Domain of $f(x)$ is $(-\infty, \infty)$.

B The y-intercept is 0. To find x-intercepts we solve $f(x) = 0$, $x - 3x^{1/3} = 0$, $x^{1/3}(x^{2/3} - 3) = 0$ which gives three solutions $x = 0$, $x = -3^{3/2}$, and $x = 3^{3/2}$.

C. $f(-x) = -x + 3x^{1/3} = -f(x)$. Hence $f(x)$ is odd. The curve $y = f(x)$ is symmetric about the origin. It is clear that $f(x)$ is a non-periodic function.

D. There is no asymptotes.

E. $f'(x) = 1 - x^{-2/3} = 1 - \dfrac{1}{x^{2/3}} = \dfrac{x^{2/3}-1}{x^{2/3}}$

$f'(x) = 0$ when $x^{2/3} - 1 = 0$ or when $x = -1$, $x = 1$,

$f'(x)$ is undefined when $x = 0$ (the graph has vertical tan. line at $x = 0$).

CNs are $x=-1$, $x=0$, and $x=1$.

The graph increases when $f'(x)>0$ or when x is in $(-\infty,-1)\cup(1,\infty)$.

The graph decreases when $f'(x)<0$ or when x is in $(-1,1)$.

F. The local maximum is $f(-1)=-1+3=2$ since $f'(x)$ changes its sign from "+" to "−".

The local minimum is $f(1)=1-3=-2$ since $f'(x)$ changes its sign from "−" to "+".

$f'(x)$ does not change its sign at $x=0$.

G. $f''(x)=\frac{2}{3}x^{-5/3}$. IP is $(0,0)$.

The graph is CD when $f''(x)<0$ or when $x<0$, CU when $f''(x)<0$ or when $x>0$.

H. To see the graph look at the solution to the problem 23 from the section 4.4.

bonus problem [5 points extra] Find the absolute maximum and absolute minimum values of $f(x)=\sin x+\cos x$ on the interval $[0,\pi/3]$.

Solution: The function is continuous and the interval is closed. We apply the Closed interval method: $f'(x)=\cos x-\sin x$. Inside the given interval $f'(x)=0$ when $x=\pi/4$. $f'(x)$ is defined everywhere. CN is $x=\pi/4$.

$f(0)=1,\ f(\pi/4)=\sqrt{2},\ f(\pi/3)=\dfrac{1+\sqrt{3}}{2}$.

The absolute maximum value is $\sqrt{2}$, the absolute minimum value is 1.

[Note: $2\sqrt{2}>1+\sqrt{3}$. Indeed, if we square both sides, then we get $8>1+2\sqrt{3}+3$. The last is true b/c $4>2\sqrt{3}$. Square both sides to see it.]