Math 0220 **Quiz 6**

Spring 2013 **S o l u t i o n s**

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1. [5 points] Find f if $f'(x) = e^x + 5(1+x^2)^{-1}$ and $f(0) = -1$.

Solution: The general antiderivative of $f'(x) = e^x + \dfrac{5}{1+x^2}$ is $f(x) = e^x + 5\tan^{-1} x + C$.

To determine C we use $f(0) = -1$: $f(0) = e^0 + 5\tan^{-1} 0 + C = 1 + 0 + C = C + 1$.

Then $C + 1 = -1$ and $C = -2$. Hence,

$$f(x) = e^x + 5\tan^{-1} x - 2$$

2. [5 points] Let A be the area of the region that lies under the graph of $f(x) = \sqrt{x}$ between 2 and 4. Using right endpoints, find an expression for A as a limit. Do not evaluate the limit.

Solution: Since $a = 2$ and $b = 4$, the width of a subinterval is $\Delta x = \dfrac{4-2}{n} = \dfrac{2}{n}$.

So $x_1 = a + \Delta x = 2 + 2/n$, $x_2 = a + 2\Delta x = 2 + 4/n$, ... ,

$x_i = a + i\Delta x = 2 + 2i/n$, ... , $x_n = a + n\Delta x = 2 + 2n/n$.

The sum of the areas of the approximating rectangles is

$$R_n = \sqrt{x_1}\Delta x + \sqrt{x_2}\Delta x + \sqrt{x_3}\Delta x + \cdots + \sqrt{x_n}\Delta x$$

$$= \left(\sqrt{2+\frac{2}{n}} + \sqrt{2+\frac{4}{n}} + \cdots + \sqrt{2+\frac{2i}{n}} + \cdots + \sqrt{2+\frac{2n}{n}}\right)\frac{2}{n}$$

$$= \frac{2}{n}\sum_{i=1}^{n}\sqrt{2+\frac{2i}{n}}$$

Then

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\sqrt{2+\frac{2i}{n}}$$

bonus problem [5 points extra] A stone was droped off a cliff and hit the ground with a speed of 80 ft/s. What is the height of the cliff? Assume for simplisity that the gravitational acceleration is 32 ft/s^2.

Solution: We consider the path of the stone as a stright line directed downward. Let 0 correspond to the initial position of the stone on the cliff, i.e. $s(0) = 0$, where $s(t)$ is the position function. Then the height of the cliff equals the displacement of the stone.

The acceleration of the stone is the gravitational acceleration: $a(t) = g = 32$ ft/s^2.

The velocity is an antiderivative of $a(t)$: $v(t) = 32t + v(0)$. The initial velocity is zero $v(0) = 0$. Then $v(t) = 32t$. The stone hits the ground when $v(t) = 80$, or $32t = 80$, or

$t = t_1 = \dfrac{80}{g} = \dfrac{80}{32} = \dfrac{5}{2}$ sec.

The position is an antiderivative of $v(t)$: $s(t) = 16t^2 + s(0) = 16t^2$.

Then the height of the cliff is

$$s(t_1) = 16 \cdot \left(\frac{5}{2}\right)^2 = 16 \cdot \frac{25}{4} = 100 \text{ feet}$$