Math 0220 **Quiz 7**

Spring 2013 **S o l u t i o n s**

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1. (a) [3 points] Evaluate the integral by interpreting it in terms of areas

$$I = \int_{-1}^{1} \left(|x| + \sqrt{1 - x^2}\right) \, dx$$

Solution:

$$I = \int_{-1}^{1} |x| \, dx + \int_{-1}^{1} \sqrt{1 - x^2} \, dx$$

The first integral is the sum of two triangles with base 1 and height 1, which is $2 \cdot \frac{1}{2} \cdot 1 \cdot 1 = 1$. The second integral is half of the area of the unit circle, which is $\frac{1}{2}\pi$. Hence,

$$I = 1 + \frac{1}{2}\pi$$

(b) [3 points] Evaluate the integral

$$I = \int_{1}^{e} \frac{4(\ln x)^3}{x} \, dx$$

Solution: Substitution: $u = \ln x$, $du = \frac{1}{4} \, dx$, $\ln 1 = 0$, $\ln e = 1$. Then

$$I = \int_{1}^{e} 4(\ln x)^3 \cdot \frac{1}{x} \, dx = \int_{0}^{1} 4u^3 \, du = u^4 \Big|_0^1 = 1 - 0 = 1$$

2. [4 points] Evaluate the integral

$$I = \int_{0}^{\pi} t \cos 3t \, dt$$

Solution: By parts: $u = t$, $du = dt$, $dv = \cos 3t\, dt$, $v = \frac{1}{3}\sin 3t$. Then

$$I = \frac{1}{3}t\,\sin 3t\Big|_0^{\pi} - \frac{1}{3}\int_0^{\pi}\sin 3t\,dt = 0 - \frac{1}{3}\int_0^{\pi}\sin 3t\,dt = \frac{1}{9}\cos 3t\Big|_0^{\pi}$$

$$= \frac{1}{9}(\cos 3\pi - \cos 0) = \frac{1}{9}(-1-1) = -\frac{2}{9}$$

bonus problem [5 points extra] Find the area of the region enclosed by the graph of the function $f(x) = \dfrac{\sin 2x}{\sin x}$ and the x-axis when x is between $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$.

Solution: $f(x) = \dfrac{\sin 2x}{\sin x} = \dfrac{2\sin x\cos x}{\sin x} = 2\cos x$

[Note: $\sin x$ is never zero inside the given interval and the cancelation is allowed.]

$\cos x = 0$ when $x = \dfrac{\pi}{2}$. Then the area is

$$A = \int_{\pi/6}^{\pi/2}\cos x\,dx - \int_{\pi/2}^{5\pi/6}\cos x\,dx$$

Because the function $\cos x$ is positive on the interval $\left(\dfrac{\pi}{6}, \dfrac{\pi}{2}\right)$ and negative on the interval $\left(\dfrac{\pi}{2}, \dfrac{5\pi}{6}\right)$. In the second case a corresponding region lies below the x-axis and its area is given by a negative integral. Hence,

$$A = \sin x\Big|_{\pi/6}^{\pi/2} - \sin x\Big|_{\pi/2}^{5\pi/6} = 1 - \frac{1}{2} - \left(\frac{1}{2} - 1\right) = 1$$