Lecture time: 12 pm   **Midterm Exam 1**

Math 0220

Spring 2015   **Name:** _____

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers when possible.

1. (5 points)   Determine the equation of the line which has a slope $m = 3$ and passes through the point $(12, 12)$. Write it in the Slope-Intercept form.

2. (5 points)   Write an expression defining y as a function of x which best describes this graph. What is the period of this function?
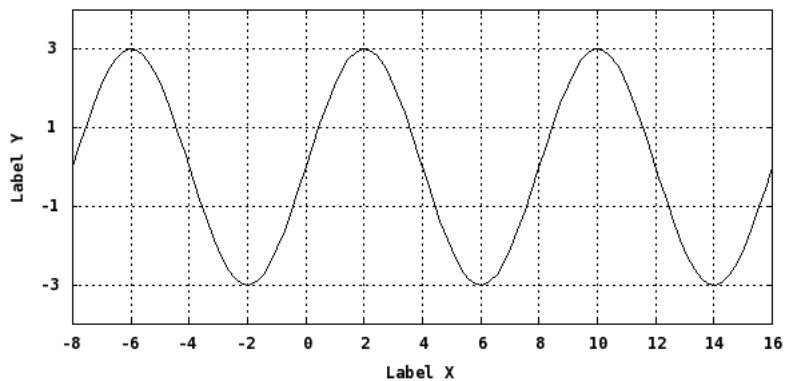
3. Find the limit, if it exists. If the limit does not exist explain why. You may use any method except the L'Hospital's Rule.

(a) (8 points) $\displaystyle\lim_{x\to 0} \frac{\sin^2(3x)}{x^2}$

(b) (8 points) $\displaystyle\lim_{x\to 0} \frac{x}{\sqrt{4-x}-2}$

(c) (8 points) $\displaystyle\lim_{x\to 0} \frac{x^2}{|x|}$.

(d) (8 points) $\displaystyle\lim_{x\to\infty} \left(x - \sqrt{x^2-x}\right)$

4. (10 points) Explain why the function

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x^2 - 2x} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

is discontinuous at $x = 2$. Find a function $g(x)$ such that $g(x) = f(x)$ when $x \neq 2$ and $g(x)$ is continuous near $x = 2$.

5. The position of a particle is given by the function $s(t) = t^2 - t + 2$

(a) (5 points) Determine the average velocity on the interval $[2, 2 + h]$. Simplify your answer.

(b) (5 points) Determine the instanteneous velocity at time $t = 2$.

6. Find the derivatives the following functions. Mention rules used. You do not need to simplify your answer.

(a) (6 points) $f(x) = 5x^2 \tan(3x)$

(b) (6 points) $f(x) = x^4(x^2 - x)(x^2 + 3x + 1)$

(c) (6 points) $f(x) = \cos^2\left(\dfrac{x^2 - 3}{x + 1}\right)$

7. (10 points)   Find an equation of the normal line to the curve $y = \dfrac{\cos^2 x}{2}$ at the point $\left(\frac{\pi}{4}, \frac{1}{4}\right)$. Write the answer in the slope-intercept form.

8. (10 points)   A spherical balloon is being pumped at a rate 8 cubic feet per minute. Determine the rate at which the radius of the balloon is changing when the diameter is 4 feet.

bonus problem (10 points extra)   Find the limit $\lim\limits_{x \to 0} \dfrac{\cos(\pi + x) + 1}{x}$ if it exists.

If it does not exist explain why. Show all work. No L'Hospital's Rule is allowed.