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1. Determine the equation of the line which has a slope $m = 3$ and passes through the point $(12, 12)$. Write it in the Slope-Intercept form.

   Solution: $y - 12 = 3(x - 12),\ \ y = 2x - 24$

2. Write an expression defining y as a function of x which best describes this graph. What is the period of this trigonometric function?

   Solution: $y = 3\sin\left(\frac{\pi x}{4}\right)$. Period is 8.

3. Find the limit, if it exists. If the limit does not exist explain why. You may use any method except the L'Hospital's Rule.

   (a) $\displaystyle\lim_{x\to 0} \frac{\sin^2(3x)}{x^2}$

   Solution: $$\lim_{x\to 0} \frac{\sin^2(3x)}{x^2} = \lim_{x\to 0} \left(\frac{\sin(3x)}{x}\right)^2 = \left(\lim_{x\to 0} \frac{\sin(3x)}{x}\right)^2 = 3^2 = 9$$

   (b) $\displaystyle\lim_{x\to 0} \frac{x}{\sqrt{4-x}-2}$

   Solution: $$\lim_{x\to 0} \frac{x}{\sqrt{4-x}-2} \cdot \frac{\sqrt{4-x}+2}{\sqrt{4-x}+2} = \lim_{x\to 0} \frac{x(\sqrt{4-x}+2)}{4-x-4}$$

   $$= \lim_{x\to 0} \frac{x(\sqrt{4-x}+2)}{-x} = \lim_{x\to 0} -(\sqrt{4-x}+2) = -(\sqrt{4}+2) = -4.$$

   (c) $\displaystyle\lim_{x\to 0} \frac{x^2}{|x|}$.

   Solution: $$\lim_{x\to 0^-} \frac{x^2}{|x|} = \lim_{x\to 0^-} \frac{x^2}{-x} = \lim_{x\to 0^-} (-x) = 0$$

   $$\lim_{x\to 0^+} \frac{x^2}{|x|} = \lim_{x\to 0^+} \frac{x^2}{x} = \lim_{x\to 0^-} x = 0$$

   Hence, $\displaystyle\lim_{x\to 0} \frac{x^2}{|x|} = 0.$

(d) $\lim_{x\to\infty}(x-\sqrt{x^2-x})$

Solution: $\lim_{x\to\infty}(x-\sqrt{x^2-x}) = \lim_{x\to\infty}(x-\sqrt{x^2-x})\cdot\frac{x+\sqrt{x^2-x}}{x+\sqrt{x^2-x}}$

$$= \lim_{x\to\infty}\frac{x^2-x^2+x}{x+\sqrt{x^2-x}} = \lim_{x\to\infty}\frac{x}{x+\sqrt{x^2-x}}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{1}{1+\sqrt{1-\frac{1}{x}}} = \frac{1}{2}.$$

4. Explain why the function

$$f(x) = \begin{cases} \dfrac{x^2-4}{x^2-2x} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

is discontinuous at $x = 2$. Find a function $g(x)$ such that $g(x) = f(x)$ when $x \neq 2$ and $g(x)$ is continuous near $x = 2$.

Solution: $\lim_{x\to2} f(x) = \lim_{x\to2}\frac{x^2-4}{x^2-2x} = \lim_{x\to2}\frac{(x+2)(x-2)}{x(x-2)} = \lim_{x\to2}\frac{x+2}{x} = 2.$

$f(2) = 1$ and $\lim_{x\to2} f(x) \neq f(2)$. Hence, the function is discontinuous at $x = 2$.

$g(x) = \frac{x+2}{x}$

5. The position of a particle is given by the function $s(t) = t^2 - t + 2$

(a) Determine the average velocity on the interval $[2, 2+h]$. Simplify your answer.

Solution: $v_{average} = \frac{s(2+h)-s(2)}{h} = \frac{(4+4h+h^2-2-h+2)-4}{h} = \frac{3h+h^2}{h} = 3+h.$

(b) Determine the instantaneous velocity at time $t = 2$.

Solution: $v(2) = \lim_{h\to0}\frac{s(2+h)-s(2)}{h} = \lim_{h\to0}(3+h) = 3.$

Alternative way: $v(t) = s'(t) = 2t-1,\quad v(2) = 4-1 = 3.$

6. Find the derivatives the following functions. Mention rules used. You do not need to simplify your answer.

(a) $f(x) = 5x^2 \tan(3x)$

Solution: $f'(x) = 10x\tan(3x) + 15x^2\sec^2(3x)$

Product, Power, and Chain Rules.

(b) $f(x) = x^4(x^2 - x)(x^2 + 3x + 1)$

Solution: $f'(x) = 4x^3(x^2 - x)(x^2 + 3x + 1) + x^4(2x - 1)(x^2 + 3x + 1) + x^4(x^2 - x)(2x + 3)$

Product and Power Rules.

(c) $f(x) = \cos^2\left(\dfrac{x^2 - 3}{x + 1}\right)$

Solution: $f'(x) = 2\cos\left(\dfrac{x^2 - 3}{x + 1}\right)\left(-\sin\left(\dfrac{x^2 - 3}{x + 1}\right)\right) \cdot \dfrac{(2x)(x+1) - (x^2 - 3)(1)}{(x+1)^2}$

Chain, Quotient, and Power Rules.

7. Find an equation of the normal line to the curve $y = \dfrac{\cos^2 x}{2}$ at the point $\left(\frac{\pi}{4}, \frac{1}{4}\right)$. Write the answer in the slope-intercept form.

Solution: The normal line equation is $y - \frac{1}{4} = -\frac{1}{m}\left(x - \frac{\pi}{4}\right)$, where $m = y'\left(\frac{\pi}{4}\right)$.

$y'(x) = \dfrac{d}{dx}\left(\dfrac{\cos^2 x}{2}\right) = \frac{1}{2} \cdot 2\cos x(-\sin x) = -\cos x \sin x$, $y'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2}$.

The normal line equation is $y - \frac{1}{4} = 2\left(x - \frac{\pi}{4}\right)$ or $y = 2x - \frac{\pi}{2} + \frac{1}{4}$.

8. A spherical balloon is being pumped at a rate 8 cubic feet per minute. Determine the rate at which the radius of the balloon is changing when the diameter is 4 feet.

Solution: $V = \dfrac{4}{3}\pi r^3$, where both V and r are functions of t. When the diameter is 4 feet, the radius is 2 feet. To find $\dfrac{dr}{dt}$ we differentiate the equation with respect to t

$\dfrac{d}{dt}\left[V = \dfrac{4}{3}\pi r^3\right]$, $\dfrac{dV}{dt} = \dfrac{4}{3}\pi \cdot 3r^2\dfrac{dr}{dt}$.

We have $r = 2$, $\dfrac{dV}{dt} = 8$. Then $8 = 4\pi \cdot 4\dfrac{dr}{dt}$ and $\dfrac{dr}{dt} = \dfrac{1}{2\pi}$ ft/min.

bonus problem Find the limit $\lim_{x \to 0} \frac{\cos(\pi + x) + 1}{x}$ if it exists. If it does not exist explain why. Show all work. No L'Hospital's Rule is allowed.

Solution: $\lim_{x \to 0} \frac{\cos(\pi + x) + 1}{x} = \lim_{h \to 0} \frac{\cos(\pi + h) - \cos \pi}{h} = f'(\pi)$, where $f(x) = \cos x$.

$f'(x) = -\sin x$ $\quad\quad \lim_{x \to 0} \frac{\cos(\pi + x) + 1}{x} = -\sin \pi = 0$