Lecture time: 1 pm

Midterm Exam 2

Math 0220

Spring 2015

Name: _____

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers when possible.

---

1. (10 points) Use linear approximation to estimate the number $\frac{1}{2.01}$.

2. (10 points) The function $f(x) = 5 + x + 2\tan^{-1} x,\ -1 < x < 1$ is one-to-one. Find $\left(f^{-1}\right)'(5)$.

3. (10 points) A bacteria culture initially ($t = 0$) contains 100 cells and grows at a rate proportional to its size. After three hours ($t = 3$) the population has increased to 500. How many cells there were two hours ($t = 2$) after the initial moment? Leave your answer in exact form.

4. (15 points) Find the point on the line $y = -2x + 5$ that is closest to the origin. Use optimization method to solve the problem.

5. Find the limit, if it exists. If the limit does not exist explain why. You may use the L'Hospital's Rule.

(a) (10 points) $\lim_{x\to 0} \dfrac{e^{2x} - 2x - 1}{x^2}$

(b) (10 points) $\lim_{\theta\to \frac{\pi}{2}} \dfrac{\sin\theta - 1}{1 - \cos 4\theta}$

(c) (10 points) $\lim_{x\to 0^+} x^{\sqrt{x}}$

6. (15 points) For the function $f(x) = 4x^3 - x^4$ make two sign diagrams: one for the first derivative that also contains information about f (CNs, increase/decrease, relative maximums and minimums), the other for the second derivative that also contains information about f (IPs, concavity).

7. (10 points) For the equation $x^2 = 6$ use Newton's method with the initial approximation $x_1 = 2$ to find the third approximation x_3 to the positive root. (Write your answer as a reduced fraction).

bonus problem (10 points extra) Simplify the expression $\sin(\cot^{-1} x)$.