Math 0220 **Quiz 1**

Spring 2015 **S o l u t i o n s**

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1. [12 pm] (a) Calculate $\dfrac{7}{24} - \dfrac{5}{36}$

Solution: $\dfrac{7}{24} - \dfrac{5}{36} = \dfrac{21}{72} - \dfrac{10}{72} = \dfrac{11}{72}$

(b) Simplify and find exact value of $\dfrac{\sqrt{75} - \sqrt{48}}{3\sqrt{3}}$

Solution: $\dfrac{\sqrt{75} - \sqrt{48}}{3\sqrt{3}} = \dfrac{\sqrt{25 \cdot 3} - \sqrt{16 \cdot 3}}{3\sqrt{3}} = \dfrac{5\sqrt{3} - 4\sqrt{3}}{3\sqrt{3}} = \dfrac{\sqrt{3}}{3\sqrt{3}} = \dfrac{1}{3}$

1. [1 pm] (a) Calculate $\dfrac{8}{45} - \dfrac{1}{12}$

Solution: $\dfrac{8}{45} - \dfrac{1}{12} = \dfrac{32}{180} - \dfrac{15}{180} = \dfrac{17}{180}$

(b) Simplify and find exact value of $2\sqrt{27} - 3\sqrt{12}$

Solution: $2\sqrt{27} - 3\sqrt{12} = 2\sqrt{9 \cdot 3} - 3\sqrt{4 \cdot 3} = 6\sqrt{3} - 6\sqrt{3} = 0$

2. [12 pm] (a) Solve the equation $5 - x^{\frac{1}{3}} = 3$

Solution: $5 - x^{\frac{1}{3}} = 3 \quad \Leftrightarrow \quad x^{\frac{1}{3}} = 2 \quad \Leftrightarrow \quad x = 8$

(b) Solve the inequality $\dfrac{2^x}{3^x} \leq \dfrac{4}{9}$

Solution: $\dfrac{2^x}{3^x} \leq \dfrac{4}{9} \quad \Leftrightarrow \quad \left(\dfrac{2}{3}\right)^x \leq \left(\dfrac{2}{3}\right)^2 \quad \Leftrightarrow \quad x \geq 2 \quad \Leftrightarrow \quad x \in [2, \infty).$

2. [1 pm] (a) Solve the inequality $5^{2x-6} < 25$

Solution: $5^{2x-6} < 25 \quad \Leftrightarrow \quad 5^{2x-6} < 5^2 \quad \Leftrightarrow \quad 2x - 6 < 2 \quad \Leftrightarrow \quad x < 4 \quad \Leftrightarrow \quad x \in (-\infty, 4).$

(b) Solve the equation $5 - x^{\frac{2}{3}} = 1$

Solution: $5 - x^{\frac{2}{3}} = 1 \quad \Leftrightarrow \quad x^{\frac{2}{3}} = 4 \quad \Leftrightarrow \quad x = 8$

3. [12 pm] (a) Prove the identity $\dfrac{\tan x}{\sec x} = \sin x$

Solution: $\dfrac{\tan x}{\sec x} = \tan x \div \dfrac{1}{\cos x} = \dfrac{\sin x}{\cos x} \cdot \cos x = \sin x$

(b) Solve the equation $\sin x \cos x = \dfrac{\sqrt{2}}{4}$. Find all solutions.

Solution: $\sin x \cos x = \dfrac{\sqrt{2}}{4} \quad \Leftrightarrow \quad \dfrac{1}{2}\sin 2x = \dfrac{\sqrt{2}}{4} \quad \Leftrightarrow \quad \sin 2x = \dfrac{\sqrt{2}}{2}$

From the unit circle we get two solutions $2x_1 = \dfrac{\pi}{4}$ and $2x_2 = \dfrac{3\pi}{4}$.

Sine is a periodic function with the period 2π. So, the actual solutions form two infinite sequences of numbers

$2x_1 = \dfrac{\pi}{4} + 2\pi k$ and $2x_2 = \dfrac{3\pi}{4} + 2\pi k$.

These two sequences of solutions can be combined in a single formula $2x = (-1)^{k-1}\dfrac{\pi}{4} + \pi k$, where k is an integer number.

Then $x = (-1)^{k-1}\dfrac{\pi}{8} + \dfrac{\pi k}{2}$.

3. [1 pm] (a) Prove the identity $(\cos x - \sin x)^2 = 1 - \sin 2x$

Solution: $(\cos x - \sin x)^2 = \cos^2 x + \sin^2 x - 2\cos x \sin x = 1 - \sin 2x$

(b) Solve the equation $\sin^4 4x - \cos^4 4x = 1$. Find all solutions.

Solution: $\sin^4 4x - \cos^4 4x = 1 \quad \Leftrightarrow \quad (\sin^2 4x - \cos^2 4x)(\sin^2 4x + \cos^2 4x) = 1$

$\Leftrightarrow \quad (-\cos(2 \cdot 4x))(1) = 1 \quad \Leftrightarrow \quad \cos 8x = -1 \quad \Leftrightarrow \quad 8x = \pi + 2\pi k,$

where k is an integer number.

Then $x = \dfrac{\pi}{8} + \dfrac{\pi k}{4}$.