Math 0220 **Quiz 2**

Spring 2015 **S o l u t i o n s**

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1. (12 pm) For the functions $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{x+1}{x+2}$

(a) [1 point] Find the function $h(x) = g \circ f$

Solution: $h(x) = \dfrac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2}$

Or $h(x) = \dfrac{x^2 + x + 1}{x^2 + 2x + 1} = \dfrac{x^2 + x + 1}{(x+1)^2}$, when $x \neq 0$.

(b) [2 points] Find the domain of $h(x)$

Solution: The domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

(c) [2 points] Evaluate the limit $\lim\limits_{x \to -1} h(x)$, if it exists or show that it does not exist.

Solution: $\lim\limits_{x \to -1} h(x) = \lim\limits_{x \to -1} \dfrac{x^2 + x + 1}{(x+1)^2}$

$\lim\limits_{x \to -1^-} \dfrac{x^2 + x + 1}{(x+1)^2} = \infty$, $\lim\limits_{x \to -1^+} \dfrac{x^2 + x + 1}{(x+1)^2} = \infty$

Hence, $\lim\limits_{x \to -1} h(x) = \infty$

1. (1 pm) For the functions $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{x+1}{x+2}$

(a) [1 point] Find the function $h(x) = f \circ g$

Solution: $h(x) = \dfrac{x+1}{x+2} + \dfrac{x+2}{x+1}$.

(b) [2 points] Find the domain of $h(x)$

Solution: The domain is $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

(c) [2 points] Evaluate the limit $\lim_{x\to -1} h(x)$, if it exists or show that it does not exist.

Solution: $\lim_{x\to -1} h(x) = \lim_{x\to -1} \frac{x+1}{x+2} + \lim_{x\to -1} \frac{x+2}{x+1} = 0 + \lim_{x\to -1} \frac{x+2}{x+1} = \lim_{x\to -1} \frac{x+2}{x+1}$

$$\lim_{x\to -1^-} \frac{x+2}{x+1} = -\infty, \qquad \lim_{x\to -1^+} \frac{x^2+x+1}{(x+1)^2} = \infty$$

Hence, $\lim_{x\to -1} h(x)$ DNE

2. (12 pm) (a) [3 points] Does the function $f(x) = \dfrac{x^3-1}{x-1}$ have a removable discontinuity at $a = 1$?

Solution: $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1} = \lim_{x\to 1} (x^2+x+1) \overset{DSP}{=} 3$

Yes, the discontinuity is removable since the limit exists.

(b) [2 points] If it does then define a continuous function $g(x)$ such that $g(x) = f(x)$ for all $x \neq 1$.

Solution: $g(x) = x^2 + x + 1$

2. (1 pm) (a) [3 points] Does the function $f(x) = \dfrac{x^4-1}{x-1}$ have a removable discontinuity at $a = 1$?

Solution: $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{(x-1)(x+1)(x^2+1)}{x-1} = \lim_{x\to 1} (x+1)(x^2+1) \overset{DSP}{=} 4$

Yes, the discontinuity is removable since the limit exists.

(b) [2 points] If it does then define a continuous function $g(x)$ such that $g(x) = f(x)$ for all $x \neq 1$.

Solution: $g(x) = (x+1)(x^2+1)$

bonus problem [5 points extra] Is there a number that exactly 1 more than its cube?

Solution: We assume that such a number exists and denote it by x.

Then the problem can be described as: Is there a solution to the equation $x^3 + 1 = x$?

In other words, is there a root of the polynomial $f(x) = x^3 - x + 1$?

$f(-2) = -5 < 0$, $f(0) = 1 > 0$ and $f(x)$ is continuous on the closed interval $[-2, 0]$.

By the Intermediate Value Theorem there is a number $c \in (-2, 0)$ such that $f(c) = 0$. So, c is a root that solves the problem.

Therefore, such a number exists.