Math 0220 **Quiz 3**

Spring 2015 **S o l u t i o n s**

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1. (12 pm) For the functions $f(x) = \cos x$ and $g(x) = \sqrt[3]{7 - x^2}$.

(a) [2 points] Find functions $F(x) = f \circ g$ and $G(x) = g \circ f$

Solution: $F(x) = \cos\left(\sqrt[3]{7 - x^2}\right)$ $G(x) = \sqrt[3]{7 - \cos^2 x}$

(b) [3 points] Find derivatives $F'(x)$ and $G'(x)$. Do not simplify results.

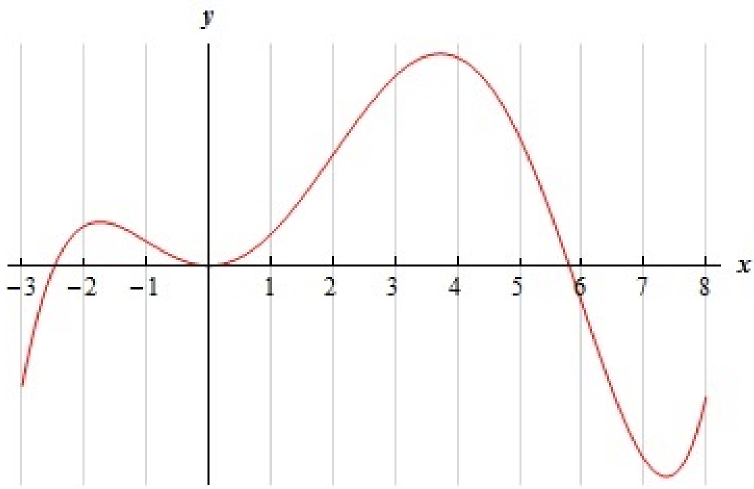
Solution: $F'(x) = \left(\cos\left((7 - x^2)^{1/3}\right)\right)' = \left(-\sin\left((7 - x^2)^{1/3}\right)\right)\left(\frac{1}{3}(7 - x^2)^{-2/3}\right)(-2x)$

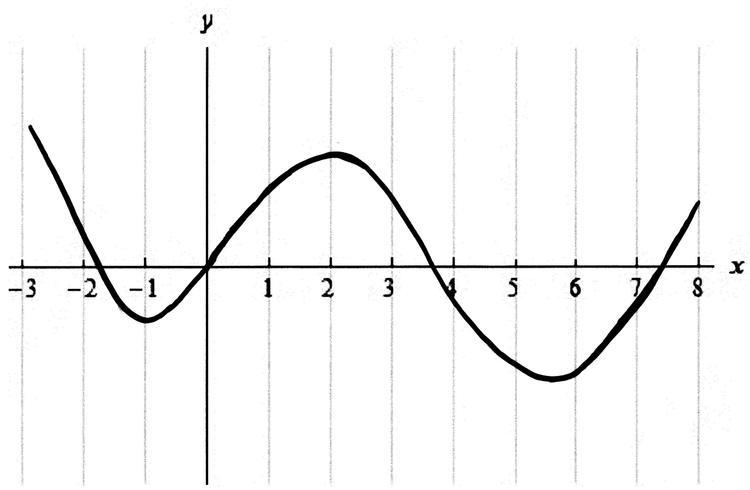
$$F'(x) = \frac{2x \sin\left((7 - x^2)^{1/3}\right)}{3(7 - x^2)^{2/3}}$$

$$G'(x) = \left(\sqrt[3]{7 - \cos^2 x}\right)' = \left((7 - \cos^2 x)^{1/3}\right)' = \frac{1}{3}(7 - \cos^2 x)^{-2/3}(-2\cos x)(-\sin x)$$

$$G'(x) = \frac{\sin 2x}{3(7 - \cos^2 x)^{2/3}}.$$

1. (1 pm) [5 points] A graph of a function is given. In the coordinate plane below plot the graph of its derivative. Mark all important points.





2. (12 pm) [5 points] Find the Slope-Intercept form of the equation of the tangent line to the curve $y = \dfrac{2}{x}$ at the point where $x = 1$.

Find the slope of the tangent line as a limit of difference quotient. No credit will be given if you do not use the limit.

Solution: Slope $m = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \dfrac{\dfrac{2}{1+h} - \dfrac{2}{1}}{h}$

$$= \lim_{h\to 0} \frac{\dfrac{2-2(1+h)}{1+h}}{h} = \lim_{h\to 0} \frac{2-2-2h}{h(1+h)}$$

$$= \lim_{h\to 0} \frac{-2h}{h(1+h)} = \lim_{h\to 0} \frac{-2}{1+h} \overset{DSP}{=} \frac{-2}{1+0} = -2.$$

$y_1 = \dfrac{2}{1} = 2$ The tangent line equation: $y - 2 = -2(x-1) \quad \Leftrightarrow \quad y = -2x + 4$

2. (1 pm) [5 points] Find an equation of the tangent line to the curve $\cos(2x+y) = y^2 \cos x$ at the point $\left(\frac{\pi}{4}, 0\right)$. Write answer in the Slope-Intercept form.

Solution: Implicit differentiation: $-\sin(2x+y)(2+y') = 2yy' \cos x - y^2 \sin x$.

When $x = \frac{\pi}{4}$ and $y = 0$ we get $-\sin\left(\frac{\pi}{2}\right)(2+y') = 0 \quad \Leftrightarrow \quad -2-y' = 0 \quad \Leftrightarrow \quad y'\left(\frac{\pi}{4}\right) = -2 = m$.

The tangent line equation: $y - 0 = -2\left(x - \frac{\pi}{4}\right) \quad \Leftrightarrow \quad y = -2x + \frac{\pi}{2}$

bonus problem (12 pm) [5 points extra] $f(x) = \sin^2 x$. Find $f^{(9)}\left(\frac{\pi}{4}\right)$. Simplify your answer.

Solution: $f'(x) = 2\sin x \cos x = \sin 2x$, $f^{(9)}(x) = 2^8 \sin 2x$, $f^{(9)}\left(\frac{\pi}{4}\right) = 2^8 = 256$.

(Hint: Use the fact that $(\sin x)^{(8)} = (\sin x)^{(4)} = \sin x$).

bonus problem (1 pm) [5 points extra] $f(x) = \cos^2 x$. Find $f^{(9)}\left(\frac{\pi}{4}\right)$. Simplify your answer.

Solution: $f'(x) = 2\cos x(-\sin x) = -\sin 2x$, $f^{(9)}(x) = -2^8 \sin 2x$, $f^{(9)}\left(\frac{\pi}{4}\right) = -2^8 = -256$.

(Hint: Use the fact that $(\sin x)^{(8)} = (\sin x)^{(4)} = \sin x$).