Math 0220 **Quiz 5**

Spring 2015 **S o l u t i o n s**

---

1. (12 pm) Find the limit, if it exists. If the limit does not exist explain why. You may use the L'Hospital's Rule.

(a) $\lim\limits_{t\to 0} \dfrac{3t + e^{-3t} - 1}{t^2}$

Solution: $\lim\limits_{t\to 0} \dfrac{3t + e^{-3t} - 1}{t^2} \underset{"\frac{0}{0}"}{\overset{H}{=}} \lim\limits_{t\to 0} \dfrac{3 - 3e^{-3t}}{2t} \underset{"\frac{0}{0}"}{\overset{H}{=}} \lim\limits_{t\to 0} \dfrac{9e^{-3t}}{2} \overset{DSP}{=} \dfrac{9}{2}$

(b) $\lim\limits_{x\to 0} (1-3x)^{\frac{2}{x}}$

Solution: Denote $y = (1-3x)^{\frac{2}{x}}$. Then $\ln y = \frac{2}{x}\ln(1-3x) = \dfrac{2\ln(1-3x)}{x}$ and

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{2\ln(1-3x)}{x} \underset{"\frac{0}{0}"}{\overset{H}{=}} \lim_{x\to 0} \frac{2\cdot\frac{-3}{1-3x}}{1} \overset{DSP}{=} \frac{2(-3)}{1} = -6$$

Therefore, $\lim\limits_{x\to 0} (1-3x)^{\frac{2}{x}} = \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} e^{\ln y} = e^{\lim\limits_{x\to 0} \ln y} = e^{-6}$

1. (1 pm) Find the limit, if it exists. If the limit does not exist explain why. You may use the L'Hospital's Rule.

(a) $\lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{1+\cos 2\theta}{1-\sin\theta}$

Solution: $\lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{1+\cos 2\theta}{1-\sin\theta} \underset{"\frac{0}{0}"}{\overset{H}{=}} \lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{-2\sin 2\theta}{-\cos\theta} \underset{"\frac{0}{0}"}{\overset{H}{=}} \lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{-4\cos 2\theta}{\sin\theta} \overset{DSP}{=} \dfrac{4}{1} = 4$

(b) $\lim\limits_{x\to 1^+} x^{\frac{x}{1-x}}$

Solution: Denote $y = x^{\frac{x}{1-x}}$. Then $\ln y = \frac{x}{1-x}\ln x = \dfrac{x\ln x}{1-x}$ and

$$\lim_{x\to 1^+} \ln y = \lim_{x\to 1^+} \frac{x\ln x}{1-x} \underset{"\frac{0}{0}"}{\overset{H}{=}} \lim_{x\to 1^+} \frac{\ln x + 1}{-1} \overset{DSP}{=} -1$$

Therefore, $\lim\limits_{x\to 1^+} x^{\frac{x}{1-x}} = \lim\limits_{x\to 1^+} y = \lim\limits_{x\to 1^+} e^{\ln y} = e^{\lim\limits_{x\to 1^+} \ln y} = e^{-1}$

2. (12 pm) Find the absolute maximum and absolute minimum values of the function $f(t) = 2\sin t + \cos 2t$ when $0 \le t \le \frac{3\pi}{4}$. Provide complete proof of your solution.

Solution: f is continuous and the interval $[0, \pi]$ is closed. So, by the Extreme Value Theorem the function attains its absolute maximum and absolute minimum values.

CNs: $f'(t) = 2\cos t - 2\sin 2t = 2\cos t - 2 \cdot 2\sin t\cos t = 2\cos t(1 - 2\sin t) = 0.$

$\cos t = 0,\ t = \frac{\pi}{2};\qquad 1 - 2\sin t = 0,\ \sin t = \frac{1}{2},\ t = \frac{\pi}{6}$

CNs are $\frac{\pi}{6}$ and $\frac{\pi}{2}$. End points are 0 and $\frac{3\pi}{4}$.

$f(0) = 1,\ f\left(\frac{\pi}{6}\right) = 1 + \frac{1}{2} = 1.5,\ f\left(\frac{\pi}{2}\right) = 2 - 1 = 1,\ f\left(\frac{3\pi}{4}\right) = \sqrt{2}\ (\approx 1.41)$

The absolute maximum value is 1.5 and the absolute minimum value is 1.

2. (1 pm) [5 points] For the function $f(x) = x^2 - 5x + 2\ln x$

(a) Find intervals on which f is increasing or decreasing.

(b) Find numbers x at which f attains local maximums and minimums.

(c) Find intervals of concavity and the inflection points.

Solution: Domain is $x > 0$

(a) $f'(x) = 2x - 5 + \frac{2}{x} = \dfrac{2x^2 - 5x + 2}{x}$

$f'(x) = 0 \iff 2x^2 - 5x + 2 = 0 \iff 2\left(x - \frac{1}{2}\right)(x - 2) = 0 \iff x = \frac{1}{2}$ or $x = 2$

$f'(x) > 0$ when $0 < x < \frac{1}{2}$ or $x > 2$, $\quad f'(x) < 0$ when $\frac{1}{2} < x < 2$

Therefore, $f(x)$ is increasing on the intervals $\left(0, \frac{1}{2}\right)$ and $(2, \infty)$; $f(x)$ is decreasing on the interval $\left(\frac{1}{2}, 2\right)$.

(b) $f(x)$ has local maximum value at $x = \frac{1}{2}$. The value is $f(1) = 0$

(c) $f''(x) = 2 - \frac{2}{x^2} = \dfrac{2(x^2 - 1)}{x^2}$

$f''(x) = 0 \iff x^2 - 1 = 0 \iff (x + 1)(x - 1) = 0 \iff x = 1$ ($x = -1$ is not in the domain)

$f(1) = -4$. IP is $(1, -4)$

$f''(x) > 0$ when $x > 1$, $\quad f''(x) < 0$ when $0 < x < 1$

Therefore, $f(x)$ is concave down on the interval $(0, 1)$ and is concave up on the interval $(1, \infty)$.

bonus problem (12 pm) Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all x. How large can $f(2)$ possibly be?

Solution: See the textbook, page 214, example 5.