Math 0220 **Quiz 6**

Spring 2015 **S o l u t i o n s**

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1. Use the Midpoint Rule with $n = 3$ to approximate the integral $\displaystyle\int_{1/4}^{7/4} \frac{x}{x+1}\,dx$

Solution: $\Delta x = \dfrac{\frac{7}{4} - \frac{1}{4}}{3} = \frac{2}{4} = \frac{1}{2}$.

The endpoints of three subintervals are $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, and $\frac{7}{4}$. Midpoints are $\frac{1}{2}$, 1, and $\frac{3}{2}$.

$$\int_{1/4}^{7/4} \frac{x}{x+1}\,dx \approx \Delta x\left[f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right)\right] = \tfrac{1}{2}\left[\tfrac{1}{3} + \tfrac{1}{2} + \tfrac{3}{5}\right] = \tfrac{1}{2}\cdot\frac{10+15+18}{30} = \tfrac{43}{60}$$

2. A particle moves along a line so that its velocity at time $t \geq 0$ is $v(t) = t^2 + 2t - 8$ meters per second. Find the displacement of the particle during the time period $0 \leq t \leq 3$.

Solution: The displacement is $s(3) - s(0) = \displaystyle\int_0^3 v(t)\,dt = \int_0^3 (t^2 + 2t - 8)\,dt$

$$= \left[\frac{t^3}{3} + t^2 - 8t\right]_0^3 = 9 + 9 - 24 = -6 \text{ meters.}$$

bonus problem In the previous problem find the total distance traveled during the time period $1 \leq t \leq 3$.

Solution: $v(t) = (t+4)(t-2) = 0 \;\Rightarrow\; t = 2$ is a turning point, $v(t) < 0$ when $1 \leq t < 2$ and $v(t) \geq 0$ when $2 \leq t \leq 3$. Hence the total distance traveled is

$$\int_1^3 |v(t)|\,dt = \int_1^2 (-v(t))\,dt + \int_2^3 v(t)\,dt = \int_1^2 (-t^2 - 2t + 8)\,dt + \int_2^3 (t^2 - 2t - 8)\,dt$$

$$= \left[-\frac{t^3}{3} - t^2 + 8t\right]_1^2 + \left[\frac{t^3}{3} + t^2 - 8t\right]_2^3 = \left[-\tfrac{8}{3} - 4 + 16 + \tfrac{1}{3} + 1 - 8\right] + \left[9 + 9 - 24 - \tfrac{8}{3} - 4 + 16\right]$$

$= 6$ meters.