Math 0220 **Quiz 7**

Spring 2015 **S o l u t i o n s**

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1. Evaluate the integral

$$I = \int_1^e \frac{5(\ln x)^4}{x}\, dx$$

Solution: Substitution: $u = \ln x$, $du = \frac{1}{x}\, dx$, $\ln 1 = 0$, $\ln e = 1$. Then

$$I = \int_1^e 5(\ln x)^4 \cdot \frac{1}{x}\, dx = \int_0^1 5u^4\, du = u^5 \Big|_0^1 = 1 - 0 = 1$$

2. Find the average value of the function $f(x) = \frac{x}{e^x}$ on the interval $[0, 2]$.

Solution: $f_{ave} = \frac{1}{2-0} \int_0^2 \frac{x}{e^x}\, dx = \frac{1}{2} \int_0^2 xe^{-x}\, dx$

By parts: $u = x$, $du = dx$, $dv = e^{-x}\, dx$, $v = -e^{-x}$.

Then $f_{ave} = \frac{1}{2} \left[-xe^{-x}\Big|_0^2 - \int_0^2 \left(-e^{-x}\right)\, dx \right] = \frac{1}{2} \left[-2e^{-2} + \int_0^2 e^{-x}\, dx \right]$

$= -e^{-2} + \frac{1}{2} \left[-e^{-x} \right]_0^2 = -e^{-2} - \frac{e^{-2}}{2} + \frac{1}{2} = -\frac{3}{2}e^{-2} + \frac{1}{2} = \frac{1}{2} - \frac{3}{2e^2} = \frac{e^2 - 3}{2e^2}.$

bonus problem Find $g'(t)$ if $g(t) = \int_{t^3}^2 \frac{\ln x}{x+1}\, dx$, $t > 0$.

Solution: By FTC, part 1: $g'(t) = 0 - \frac{\ln t^3}{t^3 + 1} \cdot (t^3)' = -\frac{9t^2 \ln t}{t^3 + 1}$