Math 0220   **Midterm Exam 1**
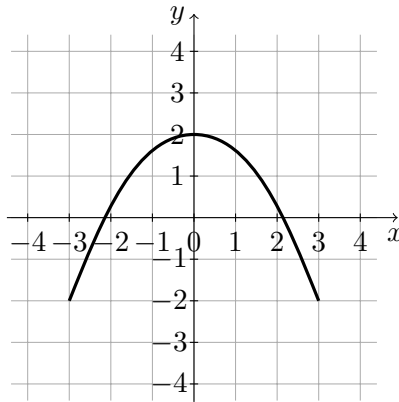
Fall 2017   **Name:**

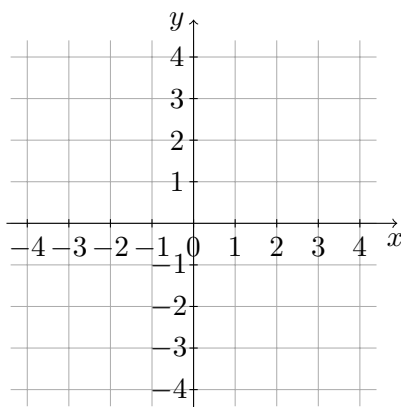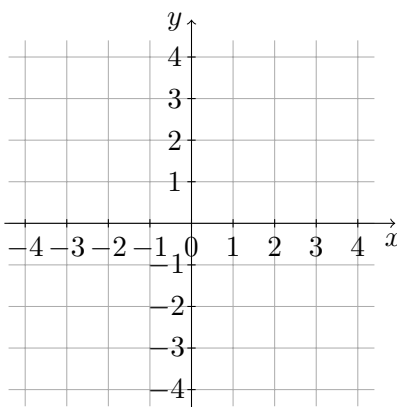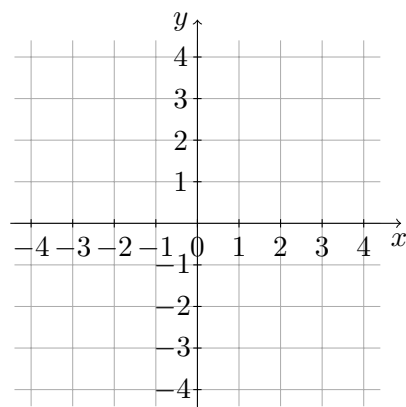No calculators, no books. Show all your work (no work = no credit).

Write neatly. Simplify your answers when possible.

1. (10 points)   The graph of $y = f(x)$ is given.

Figure 1: $y = f(x)$.

Draw $y = -\frac{1}{2}f(x+1)$ by applying three-step process:
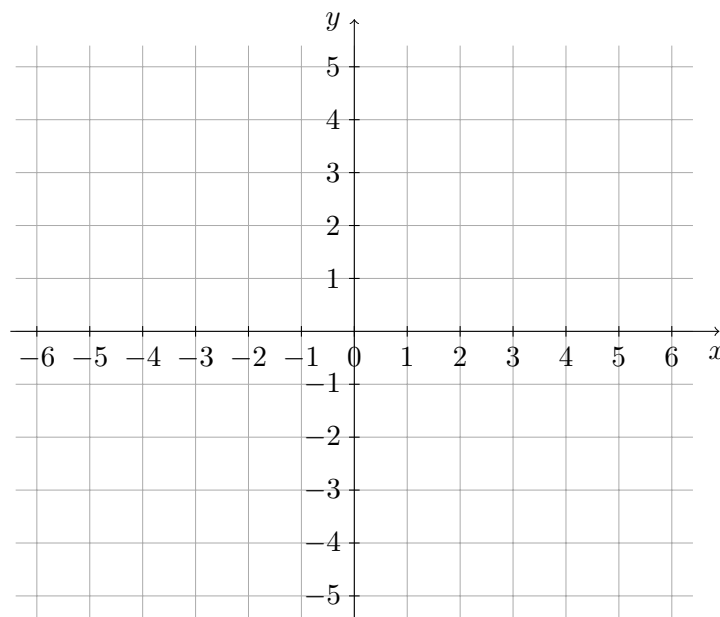
1. draw $y = \frac{1}{2}f(x)$;   2. draw $y = -\frac{1}{2}f(x)$;   3. draw $y = -\frac{1}{2}f(x+1)$.

Figure 2: $y = \frac{1}{2}f(x)$

Figure 3: $y = -\frac{1}{2}f(x)$

Figure 4: $y = -\frac{1}{2}f(x+1)$

2. (10 points)   Sketch the graph of an example of a function that satisfies all of the given conditions.

$$\lim_{x \to -1^-} f(x) = 2, \qquad \lim_{x \to -1^+} f(x) = -2, \qquad f(-1) \text{ is undefined,}$$

$$\lim_{x \to 2^+} f(x) = 1, \qquad \lim_{x \to 2^-} f(x) = 3, \qquad f(2) = -2.$$

3. (10 points)   Use the Squeeze Theorem to show that $\lim_{x \to 0} x^2 \sin\left(\frac{2}{x}\right) = 0.$

4. (10 points) Find the limit $\lim\limits_{x \to 3} \dfrac{2-x}{(x-3)^2}$.

5. (10 points) Find the derivatives of the function $f(x) = \sqrt{x-1}$ using the definition of derivative.

6.　Find the derivatives of following functions. Mention rules used. You do not need to simplify your answer.

(a) (6 points) $f(x) = 2\pi$

(b) (6 points) $f(x) = \dfrac{x^3 - 3x - 1}{\sqrt{x}}$

(c) (6 points) $g(t) = t^3 \sin t$

(d) (6 points) $h(x) = \dfrac{3x^2}{2 + x^2}$

(e) (6 points) $f(t) = \sqrt[3]{1 + \sec t}$

7. (10 points)   Find an equation of the tangent line to the curve $2x^2 + xy + y^3 = 12$

   at the point $(1,\, 2)$. Write the answer in the slope-intercept form.

8. (10 points)   Find the linearization $L(x)$ of the function $f(x) = \cos\left(x + \frac{\pi}{2}\right)$ at $a = 0$ and use it

   to approximate the number $\cos\left(\frac{\pi}{2} - 0.01\right)$.

bonus problem (10 points extra)   Find the limit $\displaystyle\lim_{x\to 0}\frac{\cos(\pi+x)+1}{\pi x}$ if it exists.

If the limit does not exist explain why. Show all work. No L'Hospital's Rule is allowed.